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Compressive Sensing Inspired Multivariate Median

Ljubiša Stanković and Miloš Daković

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Abstract A new form of the multivariate median is introduced. It is defined as a point in the multidimensional space whose sum of distances from a set of multidimensional hyperplanes is minimal. This median can be used to formulate and solve the problem of sparse signal reconstruction. Application of the proposed multivariate median is illustrated on examples.

1 Introduction

Median is an important operator in the robust statistics. It produces the solution of the norm-one minimization problem. The optimal filtering of signals (data) corrupted by an impulsive (Laplacian) noise is also obtained in a form of the median [1, 2]. In the cases of complex-valued data, a two-dimensional (complex) form of the median is used. It represents a point whose sum of norm-one or norm-two distances from the given set of complex data is minimal [3, 4]. The median is used to define robust forms of the common signal transforms [2, 5–7]. Multivariate forms of the median are defined as the point in the multidimensional space whose sum of distances from the given set of multidimensional data is minimal [8–10]. Since the multivariate median can be formulated as the solution of a norm-one minimization problem, various methods for its calculation are defined [8].

A new generalization of the multivariate median is proposed in this paper. It is inspired by the recently emerging compressive sensing formulations. The available data are considered as the parameters of the hyperplanes and the median is defined as the point with the smallest possible sum of the distances from the hyperplanes defined by the given data set. The standard median and the weighted median are the special cases of this new median, as expected.

University of Montenegro, 81000 Podgorica, Montenegro.
Email: ljubisa@ac.me and milos@ac.me.

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The new median is used to formulate and solve the sparse signal reconstruction problem from a reduced set of data. We expect that this median form can be used in other mathematical and physical problems as well.

2 Definition

Consider a P-dimensional space $\mathbb{R}^P$ and N hyperplanes in this space defined as

$$\begin{aligned} &c_{11}x_1 \ + c_{12}x_2 \ + \cdots + c_{1P}x_P = b_1 \\ &c_{21}x_1 \ + c_{22}x_2 \ + \cdots + c_{2P}x_P = b_2 \\ &\cdots \\ &c_{N1}x_1 + c_{N2}x_2 + \cdots + c_{NP}x_P = b_N, \end{aligned} \tag{1}$$

where c_{np} and b_n are the hyperplane coefficients and x_p, $p = 1, 2, \ldots, P$, are the coordinates (variables) in the considered space. In order to avoid the ambiguity in the definition of these hyperplanes we will assume

$$c_{n1}^2 + c_{n2}^2 + \cdots + c_{nP}^2 = 1 \tag{2}$$

for each n.

The weighted median is a point in the P-dimensional space $\mathbb{R}^P$ where the sum of the weighted distances between this point and hyperplanes (1) is minimal. Distance from a point to the n-th hyperplane is

$$D_n = \left|c_{n1}x_1 + c_{n2}x_2 + \cdots + c_{nP}x_P - b_n\right|. \tag{3}$$

The weighted median as a point can be obtained by minimizing the weighted sum of distances,

$$(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_P) = \underset{x_1, x_2, \ldots, x_P}{\arg\min} \sum_{n=1}^{N} d_n D_n, \tag{4}$$

where d_n are nonnegative weighting coefficients.

If all weighting coefficients are $d_n = 1$, then we will obtain a normal (non-weighted) form of the new median.

The presented median form can be considered as the norm-one minimization problem. The norm-one minimization solutions for the multivariate median are reviewed in [8]. In the examples presented in this paper we will use a method with variation of variables, using the gradient steepest-descend as in [11].

We can rewrite (1) in a matrix notation as

$$\mathbf{C}\mathbf{x} = \mathbf{b}, \tag{5}$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_P]^T$, $\mathbf{b} = [b_1, b_2, \ldots, b_N]^T$, and $\mathbf{C}$ is an $N \times P$ matrix with coefficients c_{np}. Each hyperplane is a set of points in the P-dimensional space.

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The weighted generalized median is calculated as

$$\bar{\mathbf{x}} = \text{GenWMedian}\{\mathbf{b}, \mathbf{C}, \mathbf{d}\}. \tag{6}$$

with the weights $\mathbf{d} = [d_1, d_2, \ldots, d_N]^T$.

The median form with the weighting coefficients d_n are equal to one will be referred to as the generalized median

$$\bar{\mathbf{x}} = \text{GenMedian}\{\mathbf{b}, \mathbf{C}\}. \tag{7}$$

Condition (2) is required for a geometrical interpretation of the generalized median only.

3 Special cases

3.1 Standard median

The standard weighted median is defined as

$$\bar{x}_1 = \arg\min_{x_1} \sum_{n=1}^{N} d_n \left| x_1 - b_n \right| = \arg\min_{x_1} \sum_{n=1}^{N} \left| d_n x_1 - d_n b_n \right|. \tag{8}$$

For a data set $\{b_1, b_2, \ldots, b_N\}$ it requires a set of weighting coefficients denoted by $\{d_1, d_2, \ldots, d_N\}$ for each data sample. It means that the standard, one-dimensional, weighted median is defined on the set $\{(b_1, d_1), (b_2, d_2), \ldots, (b_N, d_N)\}$. According to (4), this median is calculated as

$$\bar{x}_1 = \arg\min_{x_1} \sum_{n=1}^{N} \left| x_1 - \frac{b_n}{c_{n1}} \right|. \tag{9}$$

It is the weighted median of a set of values $b_1, b_2, \ldots, b_N$ with the weighting coefficients $1/c_{n1}$

$$\bar{x}_1 = \text{median}\left\{ \frac{b_1}{c_{11}}, \frac{b_2}{c_{21}}, \ldots, \frac{b_N}{c_{N1}} \right\}. \tag{10}$$

If hyperplane coefficients are normalized, $c_{n1} = 1$, we get the standard median form

$$\bar{x}_1 = \text{median}\left\{ b_1, b_2, \ldots, b_N \right\}. \tag{11}$$

In this case the considered space $\mathbb{R}^1$ is a line and the hyperplanes are points on the line. The classical median interpreted as a minimization problem is illustrated in Fig. 1.

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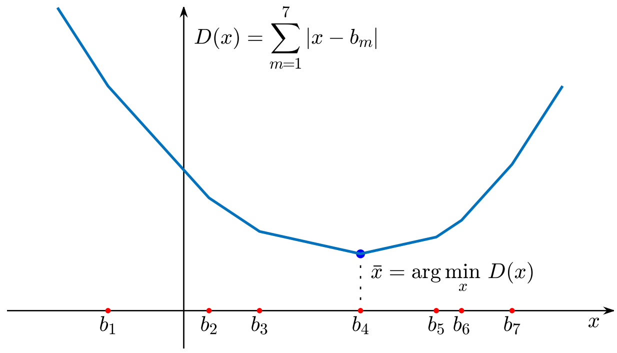

Fig. 1 One-dimensional median as a solution of the minimization problem

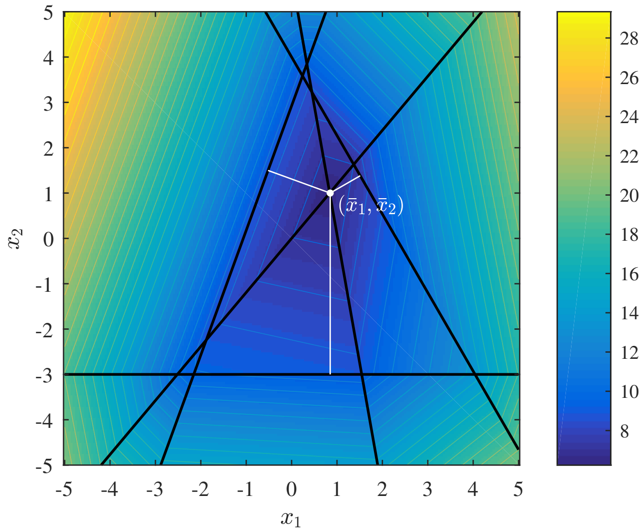

Fig. 2 Two-dimensional median as a solution of the minimization problem. The median belongs to two hyperplanes. It is marked with white dot and corresponding distances to the remaining hyperplanes are plotted with white lines.

3.2 Two-dimensional median

For $P = 2$ the equations in (1) represent the lines in a plane. Consider first the normalized case when $\sqrt{c_{n1}^2 + c_{n2}^2} = 1$ for each n. The generalized two-dimensional median can be calculated by minimizing

$$(\bar{x}_1, \bar{x}_2) = \arg\min_{x_1, x_2} \sum_{n=1}^{N} |c_{n1}x_1 + c_{n2}x_2 - b_n| \,. \tag{12}$$

The solution of this minimization problem is a point in the two-dimensional plane such that the sum of distances from this point to N given lines is min-

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imal. If the coefficients c_{n1}, c_{n2} are not normalized then we deal with the weighted generalized two-dimensional median.

The generalized two-dimensional median is calculated over the set of points belonging to the hyperplanes (lines) defined by $b_1, b_2, \ldots, b_N$ and (c_{n1}, c_{n2}).

An illustration of the generalized two-dimensional median is presented in Fig. 2.

4 Compressive sensing reconstruction formulation

Assume a K-sparse signal $x(n)$, $n = 1, 2, \ldots, N$ with the total length N. Signal values can be arranged into an $N \times 1$ vector $\mathbf{x} = [x(1), x(2), \ldots, x(N)]^T$.

Assume that M measurements are obtained as linear combination of the signal values

$$\mathbf{y} = \mathbf{A}\mathbf{x}, \tag{13}$$

where $\mathbf{y}$ is an $M \times 1$ measurement vector, $M < N$, and $\mathbf{A}$ is an $M \times N$ measurement matrix. We will also assume that the measurements are linearly independent, meaning that the rank of the measurement matrix is M.

We can split matrix $\mathbf{A}$ and vector $\mathbf{x}$ into two blocks,

$$\mathbf{A} = \begin{bmatrix}\mathbf{A}_1 & \mathbf{A}_2\end{bmatrix} \qquad \text{and} \qquad \mathbf{x} = \begin{bmatrix}\mathbf{x}_1 \\ \mathbf{x}_2\end{bmatrix}, \tag{14}$$

such that $\mathbf{A}_1$ is an $M \times P$ matrix, $P = N - M$, $\mathbf{A}_2$ is a square matrix of the order M, the length of $\mathbf{x}_1 = [x(1), x(2), \ldots, x(P)]^T$ is $P = N - M$, and the length of vector $\mathbf{x}_2 = [x(P+1), x(P+2), \ldots, x(N)]^T$ is M. Now we can rewrite (13) as

$$\mathbf{A}_2\mathbf{x}_2 = \mathbf{y} - \mathbf{A}_1\mathbf{x}_1. \tag{15}$$

Without loss of generality we can assume that matrix $\mathbf{A}_2$ is invertible. Note that the rank of the matrix $\mathbf{A}$ is M, (the measurements are independent) so there exist a subset of M linearly independent columns. These columns can always appear as the last columns of matrix $\mathbf{A}$, denoted by $\mathbf{A}_2$, with a proper reordering.

The norm-one formulation of the reconstruction problem is

$$\min ||\mathbf{x}||_1 \quad \text{subject to} \quad \mathbf{y} = \mathbf{A}\mathbf{x}. \tag{16}$$

Constraints in this minimization problem are represented by a set of M equations (13). We can consider signal values $\mathbf{x}_1$ as independent variables. The values in $\mathbf{x}_2$ can be calculated from (15) as

$$\mathbf{x}_2 = \mathbf{A}_2^{-1}\mathbf{y} - \mathbf{A}_2^{-1}\mathbf{A}_1\mathbf{x}_1 = \mathbf{b} - \mathbf{C}\mathbf{x}_1 \tag{17}$$

where $\mathbf{b} = \mathbf{A}_2^{-1}\mathbf{y} = [b_1, b_2, \ldots, b_M]^T$ and $\mathbf{C} = \mathbf{A}_2^{-1}\mathbf{A}_1$ is an $M \times P$ matrix with the elements c_{mn}.

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The reconstruction problem reduces to the minimization of norm-one of the vector

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{b} - \mathbf{C}\mathbf{x}_1 \end{bmatrix} \tag{18}$$

by varying $P = N - M$ independent signal coefficients $\mathbf{x}_1$. In a scalar form, the norm-one of this vector is

$$\begin{aligned} \|\mathbf{x}\|_1 = |x(1)| + |x(2)| + \ldots + |x(P)| + \\ |b_1 - c_{11}x(1) - c_{12}x(2) - \cdots - c_{1P}x(P)| + \\ |b_2 - c_{21}x(1) - c_{22}x(2) - \cdots - c_{2P}x(P)| + \\ \cdots \\ +|b_M - c_{M1}x(1) - c_{M2}x(2) - \cdots - c_{MP}x(P)|. \end{aligned}$$

The dependent coefficients $x(P+1), x(P+2), \ldots, x(N)$ from vector $\mathbf{x}_2$ are calculated using (17). This minimization problem can be expressed in a generalized median form. The set of hyperplanes is

$$\begin{aligned} x(1) = 0 \\ x(2) = 0 \\ \cdots \\ x(P) = 0 \\ c_{11}x(1) + c_{12}x(2) + \cdots + c_{1P}x(P) = b_1 \\ c_{21}x(1) + c_{22}x(2) + \cdots + c_{2P}x(P) = b_2 \\ \cdots \\ c_{M1}x(1) + c_{M2}x(2) + \cdots + c_{MP}x(P) = b_M. \end{aligned}$$

Independent coefficients $\mathbf{x}_1$ are obtained as the generalized median of vector

$$\mathbf{b}_0 = \begin{bmatrix} \mathbf{0}_P \\ \mathbf{b} \end{bmatrix} \tag{19}$$

with an $N \times P$ directional matrix

$$\mathbf{C}_0 = \begin{bmatrix} -\mathbf{I}_P \\ \mathbf{C} \end{bmatrix}. \tag{20}$$

Here $\mathbf{I}_P$ stands for a $P \times P$ identity matrix and $\mathbf{0}_P$ is a vector column with P zeros. Finally, the compressive sensing reconstruction problem can be formulated as

$$\mathbf{x} = \mathbf{b}_0 - \mathbf{C}_0 \,\mathrm{GenMedian}\{\mathbf{b}_0, \mathbf{C}_0\}. \tag{21}$$

The compressive sensing reconstruction procedure based on the proposed generalized median is summarized in Algorithm 1. For the generalized median calculation we can use any norm-one minimization method. For example, we can use the gradient-based minimization as in [11] that is given in Algorithm 2. Since the most of the resulting coefficients for a sparse signal will be zero-valued

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it is appropriate to assume zero-valued initial iteration (step 2). The stopping criterion (step 16) can be defined by monitoring the angle between successive gradient vectors, and the required precision (step 18) is closely related to the algorithm parameter Δ [11]. Other optimization methods that have been developed in literature can be used as well. Some of these algorithms, including the metaheuristic ones, are presented in [12, 16–22].

Algorithm 1 CS reconstruction using generalized median

Input:

– Measurements vector $\mathbf{y}$ and measurement matrix $\mathbf{A}$

1: $[M, N] \leftarrow \text{size}(\mathbf{A})$
2: $P \leftarrow N - M$
3: $\mathbf{A}_1 \leftarrow \mathbf{A}(:, 1 : P)$
4: $\mathbf{A}_2 \leftarrow \mathbf{A}(:, (P+1) : \text{end})$
5: $\mathbf{b} \leftarrow \mathbf{A}_2^{-1}\mathbf{y}, \quad \mathbf{C} \leftarrow \mathbf{A}_2^{-1}\mathbf{A}_1$
6: $\mathbf{b}_0 \leftarrow \begin{bmatrix} \mathbf{0}_P \\ \mathbf{b} \end{bmatrix}, \quad \mathbf{C}_0 \leftarrow \begin{bmatrix} -\mathbf{I}_P \\ \mathbf{C} \end{bmatrix}$
7: $\mathbf{x} \leftarrow \mathbf{b}_0 - \mathbf{C}_0\, \text{GenMedian}(\mathbf{b}_0, \mathbf{C}_0)$

Output:

– Reconstructed sparse signal $\mathbf{x}$

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Algorithm 2 Generalized median – Gradient-based minimization procedure

Input:

– Matrix $\mathbf{C}$ and vector $\mathbf{b}$

Set $m \leftarrow 0$
Set initial estimate vector $\mathbf{x}^{(0)}$ to zero
Set $\Delta \leftarrow \max_n |b(n)|$
repeat
    repeat
        $\mathbf{x}^{(m+1)} \leftarrow \mathbf{x}^{(m)}$
        for all n **do**
            $\mathbf{z}_1 \leftarrow \mathbf{x}^{(m)}$
            $z_1(n) \leftarrow z_1(n) + \Delta$
            $\mathbf{z}_2 \leftarrow \mathbf{x}^{(m)}$
            $z_2(n) \leftarrow z_2(n) - \Delta$
            $g(n) \leftarrow \|\mathbf{C}\mathbf{z}_1 - \mathbf{b}\|_1 - \|\mathbf{C}\mathbf{z}_2 - \mathbf{b}\|_1$
            $x^{(m+1)}(n) \leftarrow x^{(m)}(n) - g(n)/2$
        end for
        $m \leftarrow m + 1$
    until stopping criterion is satisfied
    $\Delta \leftarrow \Delta/3$
until required precision is achieved
$\bar{\mathbf{x}} \leftarrow \mathbf{x}^{(m)}$

Output:

– Generalized median vector $\bar{\mathbf{x}}$

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5 Numerical examples

5.1 Two-Dimensional New Median Example

Consider a set of $N = 5$ lines in the two-dimensional space ($P = 2$) shown in Fig. 2,

$$\begin{aligned} 0.9848x_1 + 0.1736x_2 &= 1 \\ 0.8660x_1 + 0.5000x_2 &= 2 \\ 0.9397x_1 - 0.3420x_2 &= -1 \\ 0.7660x_1 - 0.6428x_2 &= 0 \\ x_2 &= -3. \end{aligned} \tag{22}$$

The point whose sum of distances to these lines (hyperplanes in the two-dimensional space) is minimal is obtained as the solution of (7) with $\mathbf{b}$ and $\mathbf{C}$ in (1) defined by (22). The result produced by Algorithm 2 is

$$\bar{\mathbf{x}} = (\bar{x}_1, \bar{x}_2) = (0.85, 1). \tag{23}$$

It is shown as a white point in Fig. 2.

5.2 Compressive Sensing Higher-Dimensional Examples

Consider a measurement matrix $\mathbf{A}$ defined by

$$\mathbf{A} = \begin{bmatrix} 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 \\ 0.42 & -0.10 & -0.49 & -0.28 & 0.28 & 0.49 & 0.10 & -0.42 \\ 0.35 & -0.35 & -0.35 & 0.35 & 0.35 & -0.35 & -0.35 & 0.35 \\ 0.10 & -0.28 & 0.42 & -0.49 & 0.49 & -0.42 & 0.28 & -0.10 \\ -0.46 & -0.19 & 0.19 & 0.46 & 0.46 & 0.19 & -0.19 & -0.46 \end{bmatrix}.$$

A sparse signal $\mathbf{x}$ whose length is $N = 8$ is measured using this measurement matrix and $M = 5$ values are obtained as

$$\mathbf{y} = \begin{bmatrix} 1.995 & 0.905 & -1.995 & -1.946 & -0.133 \end{bmatrix}^T.$$

Values of the sparse signal $\mathbf{x}$ are obtained as the solution of the compressive sensing reconstruction formulation (21), with Algorithm 1, in the form

$$\mathbf{x} = \left[0, 3.2, 0, 0, 0, 2.5, 0, 0\right]^T. \tag{24}$$

This signal is sparse with sparsity $K = 2$. It is easy to check that this solution satisfies $\mathbf{Ax} = \mathbf{y}$.

In the next example we will consider a partial DCT (Discrete Cosine Transform) measurement matrix with $N = 32$ and $M = 5$ obtained by selecting rows 4, 14, 16, 17 and 22 of the full DCT matrix. A sparse signal, with sparsity $K = 2$, in the DCT domain is reconstructed using the presented algorithm from $M = 5$ measurements. The results are shown in Fig. 3. The multivariate median dimensionality is $P = N - M = 27$.

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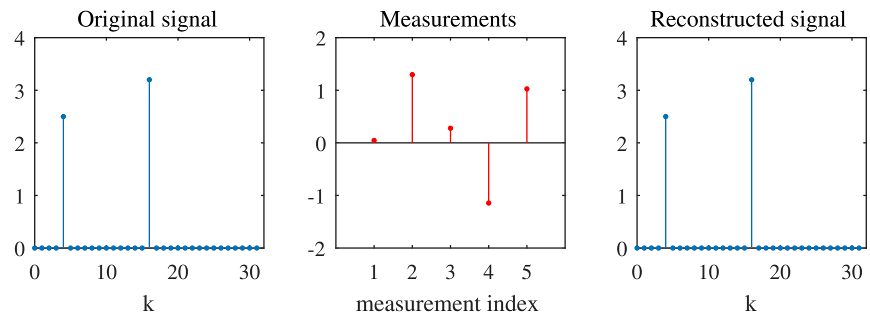

Fig. 3 Sparse signal reconstruction from $M = 5$ measurements. The original signal (left) with sparsity $K = 2$, the measurements (middle) and the reconstructed signal (right) are presented.

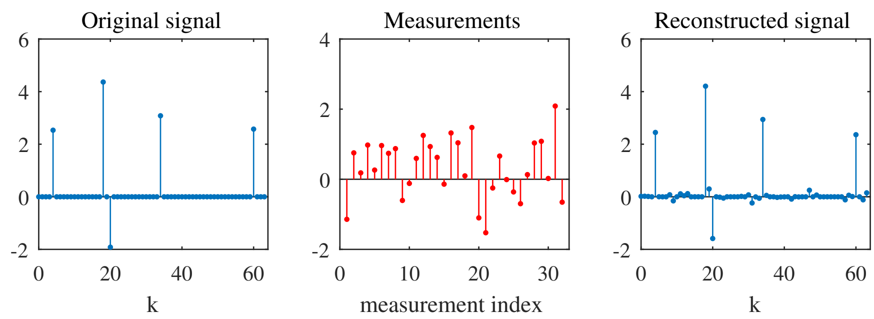

Fig. 4 Sparse signal reconstruction from $M = 32$ noisy measurements obtained with a Gaussian measurement matrix. The original signal (left) with length $N = 64$ and sparsity $K = 5$, the measurements (middle) and the reconstructed signal (right) are presented.

The Gaussian measurement matrix with $N = 64$ and $M = 32$ is considered for a sparse signal with sparsity $K = 5$. The measurements are contaminated with a Gaussian noise with the signal-to-noise ratio (SNR) 26 dB. The results are shown in Fig. 4. The SNR in the reconstructed signal is 19 dB. The multivariate median dimensionality in this example is $P = N - M = 32$.

Finally, an example with a sparse signal of length $N = 128$ and sparsity $K = 8$, sensed with $M = 64$ noisy measurements is considered. The measurement noise is heavy-tailed (cubed Gaussian) with the SNR of 17.75 dB. The signal is sparse in the DCT domain. The reconstruction is performed by using the proposed multivariate median. The results are shown in Fig. 5. The SNR in the reconstructed signal is 17.2 dB. It is approximately equal to the SNR in the measurements. Dimensionality of this multivariate median is $P = N - M = 64$.

While the median is robust to impulsive disturbances, the signals with strong outliers in the measurements should be analyzed using the methods as in [14, 15, 23].

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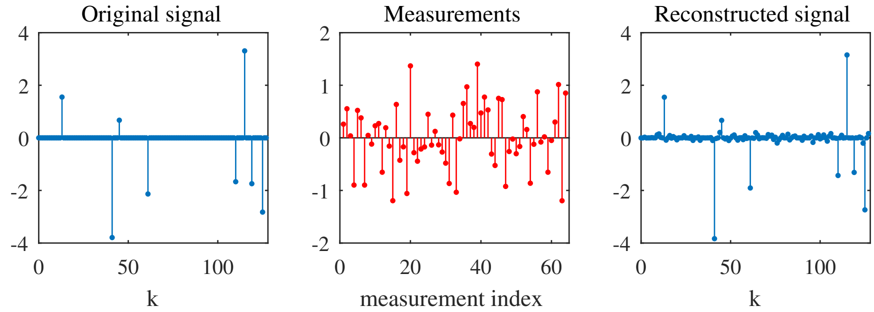

Fig. 5 Sparse signal reconstruction from $M = 64$ noisy measurements. The original signal (left) with length $N = 128$ and sparsity $K = 8$, the noisy measurements (middle), and the reconstructed signal (right) are presented.

6 Conclusion

A new multivariate form of median is introduced. This median is a point in the multidimensional space with a minimal sum of distances from a given set of hyperplanes. The new median provides a compact formulation of the compressive sensing reconstruction problem.

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