High-Resolution Local Polynomial Fourier Transform in Acoustic Signal Analysis

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***Abstract*—In this paper, we consider the separation of an acoustic signal which was transmitted through a dispersive channel. Acoustic waves transmitted through a dispersive environment are usually too complex for exact analysis. Even a simple signal can change its characteristics during the transmission. The received signal is often multi-component due to the multi-path propagation. In this paper, it is assumed that the received signal is a multi-component signal with components being very close to each other. The decomposition of the received signal using a high-resolution technique in combination with the local polynomial Fourier transform is presented. The method is numerically tested on an example.**

***Keywords*—Acoustic Signals; High-resolution; Polynomial Fourier Transform; Multi-path Signals; Dispersive Channel**

I. Introduction

The propagation of acoustic signals through a dispersive channel is a topic which challenges researches in recent years. The propagation mostly depends on the environment through which the wave is propagating. It can experience losses such as attenuation, scattering or geometrical spreading [1]. Problems can also be in the initial setup of the system which transmits and receives signals. Main problem in the analysis of such a system is that it produces nonlinear transformations of a signal [1–7]. That is, the signal changes its frequency and phase characteristics. Also, the signal is propagating with different speeds which results in nonlinear time delays at the receiver. Another problem is that a dispersive channel, because of the scattering, is usually characterized by a multi-path propagation producing multi-component signals.

Consider a signal transmitted from a transmitter to a receiver. If they are perfectly aligned, the signal which was transmitted will be received in the same form with some time shift and attenuation. The problem which we are considering is when the transmitter and the receiver are misaligned. In this case, the received signal is different from the transmitted signal. The received signal is commonly a complex, multicomponent, non-stationary signal [8–10].

The tool which is the most suitable for analysis of non-stationary signals is the time-frequency signal representation. In this paper, we will use short-time Fourier transform as a common time-frequency technique for analysis of the non-stationary signals. Specifically, we will consider its polynomial extensions, the polynomial Fourier transform (PFT) and its windowed version, local polynomial Fourier transform (LPFT) [11], [12]. Also, we assume that the transmitted signal is a linear frequency modulated (LFM) signal which was received as a two-component signal with very close components. This is why high-resolution technique proposed in [13] will be used to improve the performance of the time-frequency representations.

The paper is organized as follows. In Section II we will explain how the signal is transmitted and received from one sensor to another. The high-resolution technique used in this paper is presented in Section III and the local polynomial Fourier transform is presented in Section IV. Numerical results, as well as the conclusions, are given in Sections V and VI, respectively.

II. Modelling of the Received Signal

We will consider the transmitted signal as a LFM signal of the form

$$s_r(t) = A(t)\cos\big(2\pi(\Omega t + ct^2)\big). \tag{1}$$

where $A(t)$ is the slow varying amplitude. The parameters Ω and c are the initial frequency and the chirp rate, respectively. The signal is presented in Fig. 1.

Corresponding analytic signal $s_a(t) = s_r(t) + jHT[s_r(t)]$ is

$$s_a(t) = A(t)\exp\big(j2\pi(\Omega t + ct^2)\big) \tag{2}$$

where $HT[\cdot]$ is the Hilbert transform. The discrete signal obtained from $s_a(t)$ with sampling interval Δt is

$$s(n) = A(n\Delta t)\exp\big(j2\pi(n\Omega\Delta t + n^2c(\Delta t)^2)\big). \tag{3}$$

If the receiver is properly aligned, the signal will be received in a similar form as the transmitted one. It will just be delayed by some small time shift. The problem occurs if the receiver is not perfectly aligned [8]. In Fig. 2 the two scenarios of aligned and misaligned sensors are presented. Note that the misalignment is caused by the physical sensor positions. In Fig. 2 a) the signal is transmitted and perfectly aligned with the receiver. In Fig. 2 b), the black dashed line presents how the signal was supposed to be received and the solid line represents how the signal was actually received (misaligned with the receiver).

The physical misalignment can be due to the initial setup or the vibrations in the environment. This will cause that the signal changes through the channel since the propagation will have a dispersive nature. The received signal will then have a dispersive characteristic and it will have changes in both time

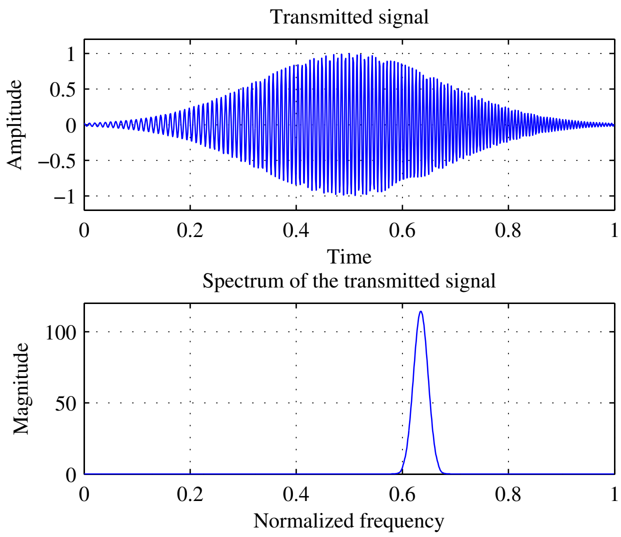

Figure 1. Transmitted signal in time-domain (top); Spectrum of the transmitted signal (bottom)

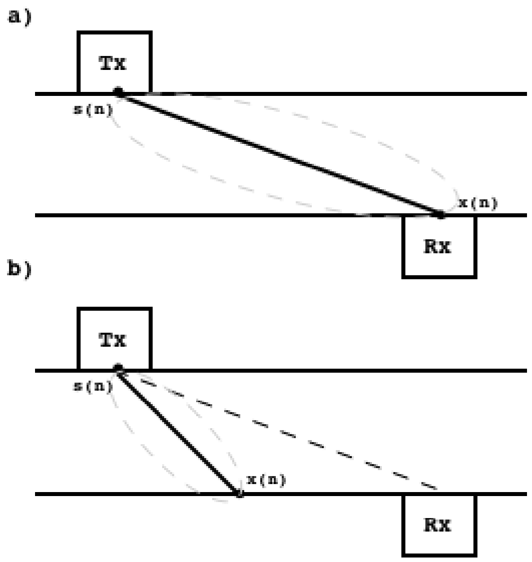

Figure 2. Physical sensor position. Examples of a) perfectly aligned receiver sensor; b) misaligned sensor. Solid line represents the received signal.

and frequency. Let us assume that the signal which is received had two propagation paths. The signal will then be:

$$x_r(n) = s_r(n) * h_1(n) + s_r(n) * h_2(n) \tag{4}$$

where $'*'$ presents the convolution of the signal $s_r(n)$ with two transfer functions $h_1(n)$ and $h_2(n)$.

Special case is when two transfer functions generate time shifted versions of the input signal with shifts t_1 and t_2.

$$\begin{aligned} x_r(t) &= s_r(t-t_1) + s_r(t-t_2) \\ &= A(t-t_1)\cos\left(2\pi(\Omega(t-t_1) + c(t-t_1)^2)\right) \\ &+ A(t-t_2)\cos\left(2\pi(\Omega(t-t_2) + c(t-t_2)^2)\right). \end{aligned}$$

Received signal consists of two components and for $t_1 \approx t_2$

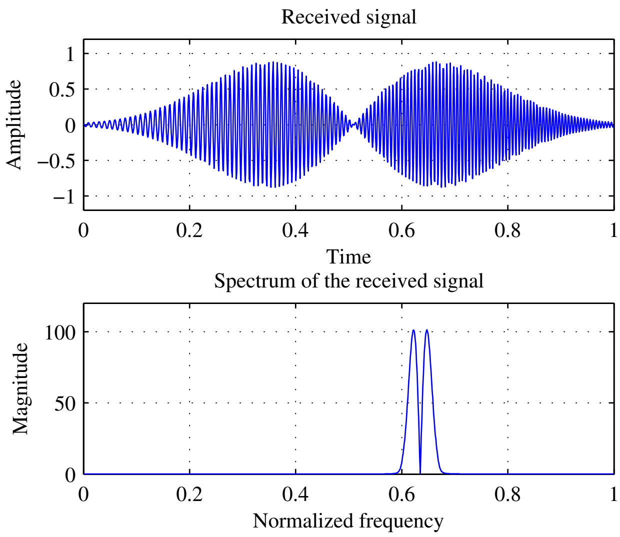

Figure 3. Received signal in a dispersive channel case (top); Spectrum of the received signal (bottom)

can be considered as modulated input signal

$$x_r(t) \approx 2A(t)\cos\left(2\pi c(t_1 - t_2)t + \phi_1\right)\cos\left(2\pi(\Omega t + ct^2)\right).$$

The received signal is shown in Fig. 3.

We can see that two received components are very close to each other in time and frequency. Our goal is to separate them so that the reconstruction of the original (transmitted) signal can be successfully done. Note that, in Fig. 3 and in the whole paper, we will neglect the signal attenuation because our main goal is to reconstruct the form of the signal. The shift in time and the attenuation which are caused during the transmission are then straightforward to find.

III. High-Resolution Decomposition

High-resolution techniques were developed for separation of close signal components. For example, they are commonly used in the estimation of DOA in array signal processing [11], [12]. There exist many high-resolution techniques used in various engineering problems such as Capon's, MUSIC, or ESPRIT. In this paper, we will consider Capon's high-resolution technique. Let us start with the basic short-time Fourier transform (STFT), which will be the key point for the signal nonstationarity analysis. The standard normalized STFT of a signal is defined as

$$STFT(\omega, n) = \frac{1}{N}\sum_{m=0}^{N-1} x(n+m)e^{-j\omega m} \tag{5}$$

which can be rewritten as

$$STFT(\omega, n) = \hat{s}_\omega(n) = \frac{1}{N}\mathbf{a}^H(\omega)\mathbf{x}(n) \tag{6}$$

where

$$\mathbf{a}(\omega) = [1, e^{j\omega}, e^{j\omega 2}, \ldots, e^{j\omega(N-1)}]^T \tag{7}$$

$$\mathbf{x}(n) = [x(n), x(n+1), \ldots, x(n+N-1)]^T. \tag{8}$$

Here $[\cdot]^T$ represents the transpose and $[\cdot]^H$ represents the Hermitian transpose.

The averaged Capon's STFT is defined as [13]

$$S_{CAPON}(n,\omega) = \frac{1}{\mathbf{a}^H(\omega)\hat{\mathbf{R}}_x^{-1}(n)\mathbf{a}(\omega)} \quad (9)$$

where

$$\hat{\mathbf{R}}_x(n) = \frac{1}{N}\sum_n \mathbf{x}(n)\mathbf{x}^H(n), \quad (10)$$

is the autocorrelation matrix over N samples (ergodicity over N samples around n is assumed), which comes from the power of the signal in the STFT representation domain.

IV. Local Polynomial Fourier Transform

Since dispersive channels are non-stationary channels, they are usually considered in the time-frequency representation domain. Many techniques for localization of time-frequency non-stationary signals in dispersive acoustic channels were developed. In this paper, we will consider both polynomial Fourier transform (PFT) and the local polynomial Fourier transform (LPFT).

The basic idea in the (L)PFT techniques is to find the parameters where the signal is maximally concentrated. Maximally concentrated distribution produces maximal transformation amplitude value as well (if the energy is preserved).

Consider a polynomial-phase signal

$$x(n) = Ae^{j(\omega n + a_2 n^2 + a_3 n^3 + \ldots + a_N n^N)}. \quad (11)$$

The polynomial Fourier transform of this signal is

$$X_{\alpha_2,\alpha_3,\ldots,\alpha_N}(\omega) = \sum_n x(n)e^{-j(\omega n + \alpha_2 n^2 + \ldots + \alpha_N n^N)}. \quad (12)$$

The PFT can be extended using a window to get the LPFT of the signal. It is defined as:

$$LPFT_{\alpha_2,\alpha_3,\ldots,\alpha_N}(\omega) = \sum_n x(n+m)w(m) \times e^{-j(\omega n + \alpha_2 n^2 + \ldots + \alpha_N n^N)}. \quad (13)$$

where $w(m)$ presents the window used for the analysis. The maximum of LPFT, i.e. the maximum of the Eq. (13) is achieved when

$$(\hat{a}_2, \hat{a}_3, \ldots, \hat{a}_N) = \arg \max_{(\omega,\alpha_2,\ldots,\alpha_N)} |LPFT_{\alpha_2,\ldots,\alpha_N}(\omega)| \quad (14)$$

where $\alpha_2, \alpha_3, \ldots, \alpha_N$ are the parameters.

Any high-resolution technique can also be applied to the LPFT [11]. In our case, we will use Capon's technique explained in the previous section. Let us consider the signal

$$x(n) = Ae^{(j(\alpha_0 n^2 + \omega_0 n + \phi_0))}. \quad (15)$$

The Capon's technique will be extended to LPFT in the sense that the autocorrelation matrix is calculated with signal multiplied by an exponential factor $\exp(-j\alpha p^2)$, i.e.

$$x_\alpha(p) = x(p)e^{(-j\alpha p^2)}. \quad (16)$$

The parameter α is obtained as the value that optimizes the concentration of

$$LPFT_\alpha(\omega,n) = \frac{1}{N}\mathbf{a}^H(\omega)\mathbf{x}_\alpha(n) \quad (17)$$

i.e., as

$$\alpha = \arg\max_\alpha |LPFT_\alpha(\omega,n)|. \quad (18)$$

For optimization we may use various concentration measures $|LPFT_\alpha(\omega,n)|$, like for example $\alpha = \arg\min_\alpha ||LPFT_\alpha(\omega,n)||_1$. Note that the standard LPFT is used in the optimization, rather than its highly concentrated version, since highly concentrated transformations are biased in amplitude and would not be appropriate for concentration comparison for different α.

The local autocorrelation function is then calculated with optimal parameter α, using a sliding window function. It is defined as

$$\hat{\mathbf{R}}_x(n,K,\alpha) = \frac{1}{K+1}\sum_{p=n-K/2}^{n+K/2} \mathbf{x}_\alpha(p)\mathbf{x}_\alpha{}^H(p) \quad (19)$$

where K is the parameter defining the width of a symmetric sliding window. The optimal local Capon's representation is

$$LPFT_{CAPON}(n,\omega) = \frac{1}{\mathbf{a}^H(\omega)\hat{\mathbf{R}}_x^{-1}(n,K,\alpha)\mathbf{a}(\omega)}. \quad (20)$$

An example will be given in the next section.

V. Numerical Results

We will consider a LFM signal to be transmitted, of the form (3). The frequency range f is between $f_{min} = 98$ Hz and $f_{max} = 138$ Hz. The sampling frequency is $f_s = 1024$ Hz. The signal was decomposed using the standard STFT, Capon's STFT, standard LPFT and the Capon's LPFT. We will use rectangular window of length $L = 128$ for the LPFT calculation. The decomposition of the signal using different methods is shown in Fig. 4. In Fig. 5, one time-instant of Fig. 4 is shown. We can see that the two components are distinguishable in the Capon's LPFT representation method.

From Fig. 4 we can conclude that two components of the analyzed signal are separated by Capon's LPFT only, while other methods force modulated single component representation of the analyzed signal.

VI. Conclusions

In this paper, a high-resolution technique for separation of components is examined. The high-resolution technique was extended to the local polynomial Fourier transform to give a better result for the separation. It is assumed that a one-component LFM signal was transmitted through a dispersive environment and that the received signal was falsely received because of the misaligned sensors. The received signal consists of two components which are very close to each other. It is shown that the other techniques such as the standard STFT, and the Capon's STFT failed to separate the two components. The method of Capon's high-resolution techniques combined with LPFT showed good results.

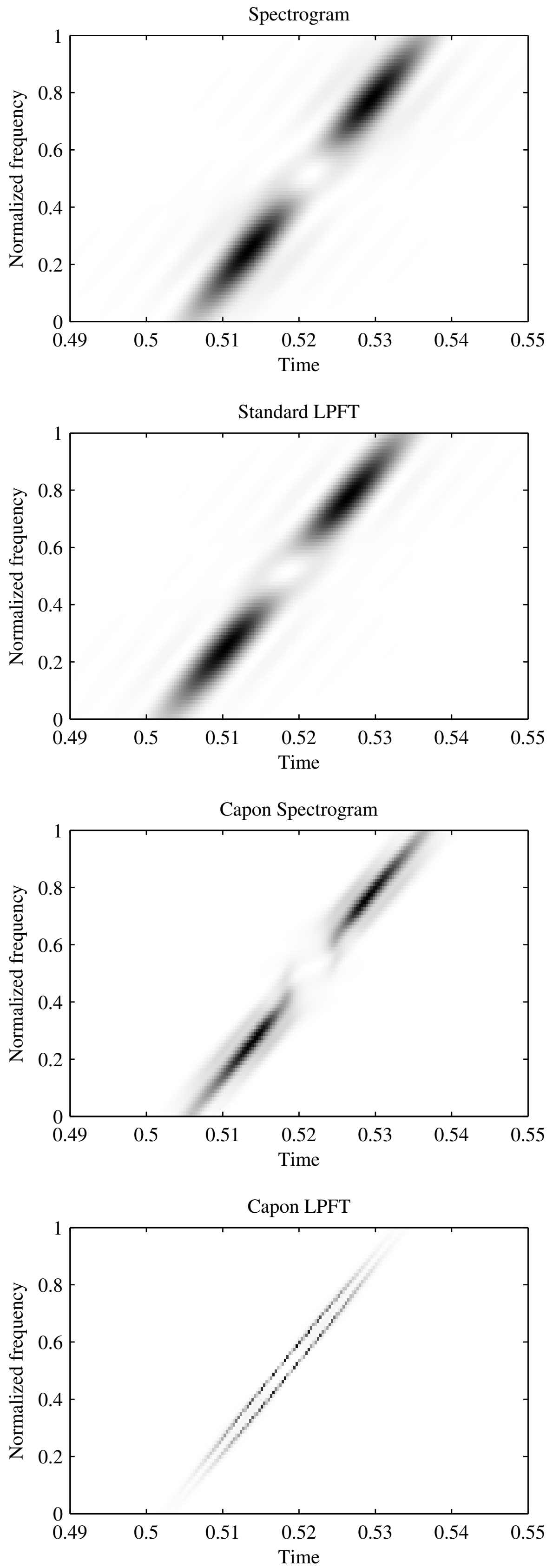

Figure 4. Decomposition using: standard STFT (first), standard LPFT (second), Capon's STFT (third), and Capon's LPFT (fourth)

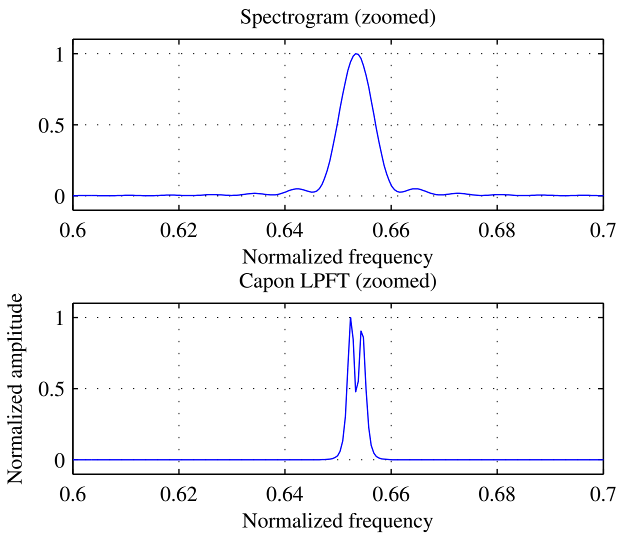

Figure 5. Zoomed spectrum of one time-instant in the standard STFT (top) and spectrum of one time-instant in the Capon's LPFT (bottom)

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