FHSS signal sparsification in the Hermite transform domain

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Abstract — **Signal sparsity is exploited in various signal processing approaches. The applicability ranges from compression, signal classification, coding, etc. Finding a suitable basis where the signal exhibits a compact (sparse) support is a challenging task and the result mainly depends on the signal nature. In this paper, we observed sinusoidally modulated signals appearing in wireless communications, namely the FHSS signals. As a sparsity domain, the Hermite transform domain is considered. The Hermite basis functions resemble the shapes of the FHSS signal components, and therefore these are considered as suitable for compact representation. In order to improve the sparsity of the observed signal components, we propose to employ a procedure for the Hermite transform optimization. As a result, the discrete Hermite functions better fit the signal components, producing just negligible errors between the original and optimized signal. The theory is verified by the experimental results. The procedure is tested on synthetic FHSS signal.**

Keywords — **FHSS signals, signal sparsity, sparsification, Hermite transform domain.**

I. Introduction

Sparse signal representation in certain domain is desirable property in signal processing and analysis [1]-[7]. Sparse signals are characterized by the condensed information of interest concentrated into a few signal coefficients [1],[6]. In other words, sparsity means that the signal energy is concentrated within small number of coefficients in the domain of sparsity [5],[6]. Many compression algorithms, such as MPEG and JPEG, exploit the fact that the signal is sparse in a certain domain, to remove redundancy and compress the signal [5]. Sparsity property is exploited recently to design a new signal strategy known as the Compressive Sensing (CS) [2]-[7]. CS strategy enables signal reconstruction from much smaller number of available signal samples (i.e. measurements), compared to the traditional sampling based on the sampling theorem. Recall that the traditional approach requires sampling with frequency at least two times higher than the maximal signal frequency. For signals in many real applications, sampling in such way results in a large number of samples. Furthermore, the optimization methods and algorithms for recovering missing information in sparse signals are constantly developing.

Signal can be sparse in time, frequency, time-frequency domain, space domain, etc. Sparse representation in certain domain can be achieved by choosing an appropriate transform basis, i.e. by decomposing the signal onto the set of suitable expansion functions. Depending on the signal nature, sparsifying basis can be wavelet transform basis, discrete Fourier transform (DFT), discrete cosine transform (DCT), Hermite transform (HT), time-frequency representation [1],[2],[5], etc.

Our focus in this paper is on the frequency hopping spread spectrum (FHSS) signals and their optimal representation in the HT domain [8]-[18]. The reasons for choosing HT are as follows. The HT founds usage in various signal processing applications – biomedical signals analysis (EEG), ultra wideband (UWB) and communication signals analysis, in computer tomography, etc. It has many desirable properties such as good computational localization in both, signal and transform domains. Also, it is applicable in compression algorithms since many signals can be modeled by using smaller number of Hermite functions, compared to the signal length [12], [13]. It provides better compression for the certain class of signals, compared to the widely used orthogonal signal transforms, such as DFT, DCT and discrete wavelet transform (DWT), as it was proved in [13]. Here we have observed signals that appear in wireless communications and described the procedure for their sparsification in the HT domain. Finding suitable domain where signal is sparse is of particular importance for application of CS approach, in order to minimize the number of samples required for signal analysis.

Particularly, in this paper the signals used in Bluetooth communications are considered and these are known as the FHSS modulated signals [8],[10],[11],[19]. Having in mind that the FHSS signals consist of short duration sinusoidal components, it is not an easy task to provide a domain with sparse representation. For instance, the commonly used DFT does not provide sparse representation of the FHSS signals. This is caused due to the spectrum leakage around the frequencies of the signal components. However, the Hermite expansion functions show similar shape as the FHSS signal components. Therefore, these functions are used to fit the signal components and to concentrate the representation in the HT domain. To enhance the signal sparsity, the optimization of the time-scaling factor and time-shift parameter of the Hermite basis functions is employed [12].

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The paper is organized as follows: Section II is the theoretical background on the HT. The procedure for the HT optimization is described in Section III, while the experimental results are given in the Section IV. Conclusion is given in the Section V.

II. Theoretical Background

The Hermite functions are closely related with widely known Hermite polynomials:

$$H_M(t) = (-1)^M e^{t^2}\left(d^M(e^{-t^2})/dt^M\right). \quad (1)$$

In terms of Hermite expansion, a continuous-time signal $f(t)$ can be represented as [5], [13], [17]:

$$f(t) = \sum_{p=0}^{\infty} c_p \psi_p(t), \quad (2)$$

where $\psi_p(t)$ denotes Hermite functions, while c_p denotes Hermite expansion coefficients. If the signal $f(t)$ as well as Hermite basis functions are sampled such that they have M discrete values available at the roots of the M-th order Hermite polynomial (1), expansion (2) becomes finite, with upper bound equal to $M-1$. If signal is sampled uniformly, values at the points of interest can be obtained incorporating interpolation techniques. Discrete Hermite function of order p is defined as [9], [13]-[18], [20]:

$$\psi_p(t_m) = \frac{e^{-t_m^2} H_p(t_m/\sigma)}{\sqrt{\sigma 2^p p! \sqrt{\pi}}}, \quad (3)$$

and also with following recursion [5]:

$$\psi_0(t_m) = \frac{1}{\sqrt[4]{\pi}} e^{-\frac{t_m^2}{2\sigma^2}}, \quad \psi_1(t_m) = \frac{\sqrt{2}t}{\sqrt[4]{\pi}} e^{-\frac{t_m^2}{2\sigma^2}},$$
$$\psi_p(t_m) = t_m\sqrt{\frac{2}{p}}\psi_{p-1}(t_m) - \sqrt{\frac{p-1}{p}}\psi_{p-2}(t_m), \ \forall p \geq 2. \quad (4)$$

The scaling factor σ is used to match functions to the signal, by stretching or compressing them. Hermite coefficients are calculated using the Gauss-Hermite quadrature expansion [5], [12]:

$$c_p \approx \frac{1}{M}\sum_{m=1}^{M} \frac{\psi_p(t_m)}{[\psi_{M-1}(t_m)]^2} f(t_m), \quad (5)$$

To summarize, the discrete HT is defined as follows:

$$\mathbf{c} = \mathbf{H}\mathbf{f},$$

$$\begin{bmatrix} c_0 \\ c_1 \\ \dots \\ c_{P-1} \end{bmatrix} = \frac{1}{M}\begin{bmatrix} \frac{\psi_0(t_1)}{(\psi_{M-1}(t_1))^2} & \dots & \frac{\psi_0(t_M)}{(\psi_{M-1}(t_M))^2} \\ \frac{\psi_1(t_1)}{(\psi_{M-1}(t_1))^2} & \dots & \frac{\psi_1(t_M)}{(\psi_{M-1}(t_M))^2} \\ \dots & \dots & \dots \\ \frac{\psi_{M-1}(t_1)}{(\psi_{M-1}(t_1))^2} & \dots & \frac{\psi_{M-1}(t_M)}{(\psi_{M-1}(t_M))^2} \end{bmatrix}\begin{bmatrix} f(t_1) \\ f(t_2) \\ \dots \\ f(t_M) \end{bmatrix}, \quad (6)$$

where $\mathbf{c}$ and $\mathbf{f}$ are Hermite coefficients and signal vectors, $\mathbf{H}$ is the transform matrix. The inverse transform reads:

$$\mathbf{f} = \mathbf{H}^{-1}\mathbf{c},$$

$$\mathbf{H}^{-1} = \begin{bmatrix} \psi_0(t_1) & \cdots & \psi_{M-1}(t_1) \\ \psi_0(t_2) & \cdots & \psi_{M-1}(t_2) \\ \vdots & \vdots \ \ddots & \vdots \\ \psi_0(t_M) & \cdots & \psi_{M-1}(t_M) \end{bmatrix}, \quad (7)$$

with $\mathbf{H}^{-1}$ being the inverse HT matrix. Note that the columns of this matrix are consisted of corresponding Hermite basis functions.

III. Hermite Transform in FHSS Signal Analysis

A. Spread Spectrum in wireless communications

In this paper, we have observed one specific type of spread spectrum (SS) modulated signal. SS techniques are developed and used for securing the data transmission. It is used for effectively securing signals in wireless communications as well. The theory behind the SS has been known since the beginning of the 20th century. The SS technique found a practical application during the World War I, when it was used by the German military. It is robust to inter-symbol interference (ISI), jamming, noise and other environmental factors. There are two types of SS modulations [8],[10],[11],[19]:

- direct sequence spread spectrum (DSSS), where fast pseudorandom sequence causes phase transitions in the carrier data. This modulation type is used in IEEE 802.11b standard for wireless LAN;
- frequency hopping spread spectrum (FHSS), where carrier is caused to shift the frequency in a pseudorandom way [19]. This modulation type is used in Bluetooth standard.

Our focus in this paper is on the FHSS signals. FHSS modulation technique uses pseudorandom sequence to determine frequencies on which parts of the signal appear. Unless the pseudorandom sequence is known, it is hard to assume the frequency at which a carrier wave will appear next. Therefore, this technique is robust to different environmental factors such as noises, nearby RF signals, etc.

Achieving the sparse or compact support of FHSS signals is the main motivation behind this work. Having in mind the shape of the considered signal components, the Hermite functions are chosen as a starting basis. Namely, the Hermite functions resemble the shape of FHSS components. However, in order to achieve better sparsity, it is necessary to adapt each basis function to the particular signal shape. Each component is firstly sampled at the points proportional to the roots of Hermite polynomial. Then, the procedure for fitting width of the Hermite functions to the width of the signal components, as well as their time shift is applied. By choosing the suitable fitting parameters, the HT can be optimally sparsified, even in the cases when the non-parameterized transform is not inherently sparse.

B. Optimal signal representation in the HT domain

The discrete signals of length M, being represented by the HT, should be sampled at non-uniform points being proportional to the roots of the M-th order Hermite polynomial. As the signals are usually sampled uniformly according to the sampling theorem, the following *sinc* interpolation formula [12] is used, for obtaining the values at the requested non-uniform points:

$$f(\lambda t_m) \approx \sum_{n=-K}^{K} f(n\Delta t)\frac{\sin\left(\pi(\lambda t_m - n\Delta t)/\Delta t\right)}{\pi(\lambda t_m - n\Delta t)/\Delta t}, \tag{8}$$

where m = 1,…, M, n = -K,…, K and Δt is the sampling period. Instead of stretching and compressing the basis functions, alternatively, we can fix $\sigma = 1$ in (3) and introduce the signal time-axis scaling factor λ. As the aim is to find the value of the parameter producing the best possible concentration (i.e. sparsity), concentration measure, namely the l_1-norm, is used as the optimization criterion:

$$\lambda_{opt} = \min_{\lambda}\left\|\tilde{\mathbf{c}}\right\|_1 = \min_{\lambda}\left\|\mathrm{HT}\{f(\lambda t_m)\}\right\|_1 = \\ = \left\|\mathrm{HT}\left\{\sum_{n=-K}^{K} f(n\Delta t)\frac{\sin\left(\pi(\lambda t_m - n\Delta t)/\Delta t\right)}{\pi(\lambda t_m - n\Delta t)/\Delta t}\right\}\right\|_1, \tag{9}$$

where the operator $\mathrm{HT}\{\cdot\}$ is used to denote the Hermite transform of the signal rescaled calculated according to (6) where $\sigma = 1$ is assumed in the definition of basis functions (3). Note also that $\left\|\tilde{\mathbf{c}}\right\|_1$ is used to denote the l_1 –norm of the Hermite coefficients $\tilde{\mathbf{c}}$ of the rescaled signal, calculated as:

$$\left\|\tilde{\mathbf{c}}\right\|_1 = \sum\nolimits_{p=0}^{M-1}\left|\tilde{c}_p\right|, \tag{10}$$

used as a measure of the HT sparsity.

The optimization problem (9) is a 1-D search over the possible values of the scaling factor λ. It was shown that the considered l_1-norm exhibits convexity under conditions considered in detail in [12], where an adaptive iterative algorithm is also proposed to solve (9) without a direct search approach.

In similar way, after the extraction of FHSS localized components $f_i(n\Delta t)$, instead of $f_i(n\Delta t)$ shifted signals $f_i((n \pm \Delta)\Delta t)$ can be used in (8), with $\Delta \in \left[-\Delta_{\max}, \Delta_{\max}\right]$ being a small integer shift left or right. For every possible Δ optimization (9) is done, and a measure vector $\mathbf{M}$ is formed. After that, the shift producing the minimal concentration measure is selected, according to:

$$\Delta = \arg\min_{\Delta} \mathbf{M}. \tag{11}$$

IV. Experimental Results

The model of the FHSS signal consisted of three components (hops) is observed. This synthetic model can be described by:

$$f(n) = \sum_{i=1}^{K} A_i e^{-\left(\frac{n-\tau_i}{\beta_i}\right)^2}\cos\left(\alpha_i n\right), \tag{12}$$

with K = 3, $\alpha_i \in \{1.4\pi/4, 0.91\pi/4, 1.25\pi/4\}$, $\beta_i = 10$, $A_i = 1$ and $\tau_i \in \{M/16, -M/15, -M/16\}$ for i = 1, 2, 3, respectively.

The total signal length is M = 100 samples. The hops have the same duration, while differing in frequency. Also, components are shifted from the origin for τ_i. The time domain as well as the DFT and the HT of the signal are shown in Fig. 1. It can be seen that the HT better concentrates the signal than the DFT. However, in order to enhance the concentration in the HT domain and further reduce the number of Hermite coefficients by using (9) and (11), signal is firstly decomposed [10], and the separated components are further considered. The shape resemblance of the separated components with the Hermite basis functions is an indication of potential sparsity representation and possible application of the approach [12]. This is confirmed by the results shown in Figs 2 and 3, illustrating the fact that the HT exhibits a better concentration when compared to DFT, and it is further improved by applying the parameters obtained by minimizations (9) and (11).

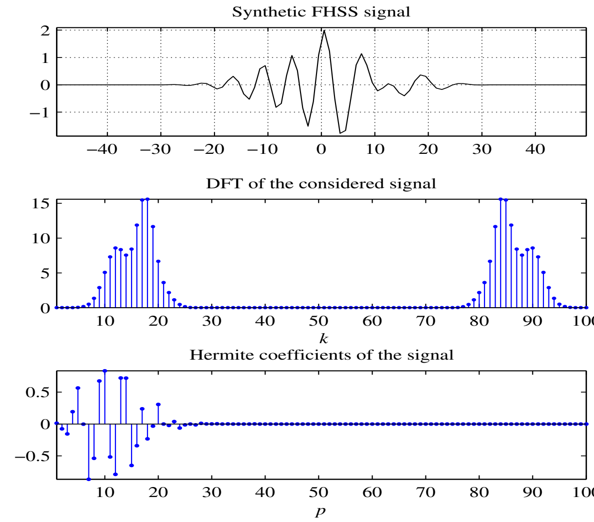

Fig. 1. The considered three-component FHSS signal: first row - time domain, second row – DFT, third row – HT domain

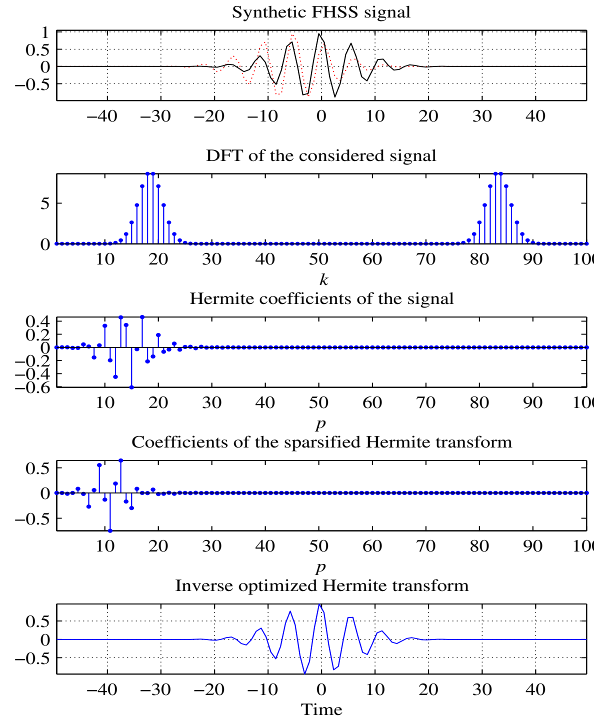

Fig. 2. The first component in the FHSS signal shown in Fig.1: original (1st row, full line) and optimally shifted component (1st row, dotted red line), DFT, HT and optimized HT (2nd – 4th rows), component reconstructed from the optimized HT domain (5th row)

Optimization of the transform basis parameters improves the sparsity and produces negligible error between original and sparsified version of the signal component (Fig. 2, 5th row, and Fig. 3, 5th and 10th rows).

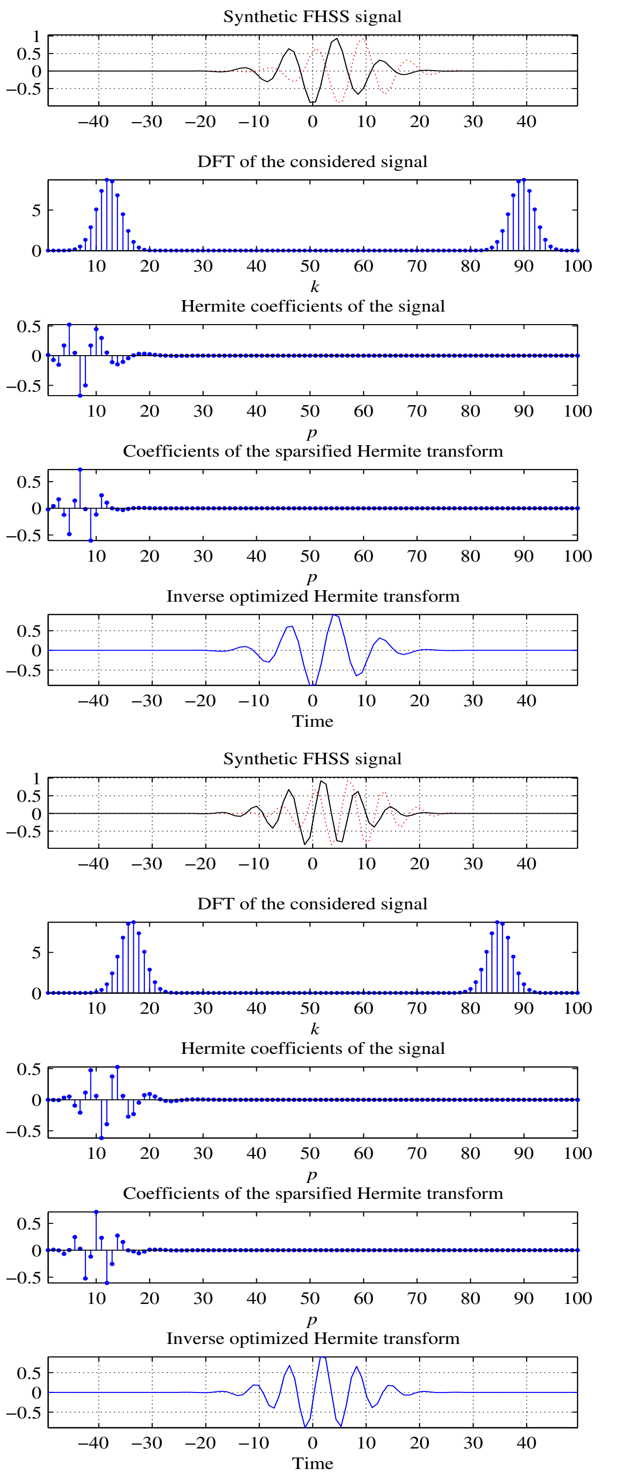

Fig. 3. The second and the third component of the FHSS signal defined in (12): original (1st and 6th rows, full line) and optimally shifted components (1st and 6th rows, dotted red line), DFT (2nd and 7th row), HT (3rd and 8th row) and optimized HT (4th and 9th row), component reconstructed from the optimized HT domain (5th and 10th row)

V. Conclusion

Representation of FHSS communication signals in the HT domain is considered. This particular transform is studied as a potential domain of sparsity of FHSS signals. Separated components of the signal are observed. The procedure for sparsification of the component's HT is proposed, and it is done by minimizing the l_1 –norm based concentration measure. The results indicate further applicability in compressed sensing scenarios.

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