Reconstruction of Global Ozone Density Data using a Gradient-Descent Algorithm

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***Abstract*—The Ozone density and its decrease in the atmosphere are one of the main concerns of nowadays. The Ozone density is tracked using the Ozone Measurements Instrument (OMI). During tracking and collection of information, missing data occur because of the pathway the instrument is going. Reconstruction of those missing data can be done using the results from the compressive sensing (CS) approach. The application of a CS algorithm based on the gradient-descent method for the Ozone density data reconstruction is presented in this paper. The algorithm is based on varying the missing data to promote sparsity in each frame. The algorithm is modified for a more efficient reconstruction by using the dynamic information about previous frames. The reconstruction results for some recent Ozone data are presented.**

***Keywords*—compressive sensing, ozone data, gradient algorithm, reconstruction**

I. Introduction

The Shannon-Nyquist theorem is used for representing continuous-time signals into discrete-time domain. It states that every signal can be recovered from the discrete-time samples if the frequency of sampling is at least twice as high as the maximum frequency of the given signal. The sampling theorem can be restated in the following way. If the Fourier transform of a continuous-time signal is of finite duration, then we do not need to know all its values in order to reconstruct the signal.

Compressive sensing (CS) is a field dealing with sparse signals. A signal can be transformed to different domains with a possibility to obtain the exact signal via the inverse transform. A signal is sparse in a transformation domain if the number of non-zero coefficients in that domain is much fewer than the number of signal samples. For a signal that has a small number of non-zero coefficients in a transformation domain we can expect that it can be recovered from a smaller set of signal samples. The goal of compressive sensing in signal processing is to reconstruct all values of a sparse signal with a smaller number of randomly positioned samples/measurements than it is required by the Shannon-Nyquist theorem if the recovery conditions are met [1–6]. Since many signal processing problems are dealing with signals which are sparse in a transformation domain, the CS already expanded in many areas, such as image processing, tracking, biomedicine, communications, radar processing, etc. Since the introduction of CS, there are many methods and techniques developed for the reconstruction of a sparse signal. The one considered in this paper belongs to a large group of the gradient-based algorithms. This algorithm uses the missing values as variables and reconstructs the unavailable signal samples/measurements [7], [8]. The idea of sparse signals reconstruction by varying the missing samples is applied in this paper to the Ozone data. The missing data occur due to a different pathway of the Ozone Measurements Instrument (OMI) in comparison to the Earth's rotation [9], [10]. The reconstruction of missing Ozone data, implicitly assuming data sparsity in the discrete-cosine transform (DCT) domain, is done in this paper. Dynamic properties of the successive Ozone data frames are used to improve calculation complexity and efficiency of the reconstruction algorithm.

The paper is organised as follows. The problem formulation in the compressive sensing approach, the basic theory of Ozone density data and the relation to dynamic signal presentation are given in the Section II. The reconstruction algorithm is presented in Section III. The results of the reconstructed data of Ozone density are shown in Section IV. The conclusions are given in Section V.

II. Theoretical Background

The general compressive sensing problem formulation is reviewed in this section, along with the theory of the Ozone density and its relation to the sparse signal processing with missing data.

A. Ozone Problem Formulation

The Ozone is a very thin layer in the stratosphere, which takes only a fifth of 1% of the Earth's atmosphere. Its role is to preserve Earth from Sun's radiation of ultraviolet waves. The Ozone density is calculated in Dobson Units (DU) and it can be defined as the amount of Ozone squeezed in 1mm and at freezing point 0 degree Celsius. The usual amount of Ozone is about 300 DU. Region where the Ozone layer is, for example, around 100 DU (thinner than usual), is defined as the Ozone hole in the atmosphere, [9–11].

The Ozone Measurements Instrument is used for tracking the Ozone density [9]. Because of the rotation of the Earth and the path of the instrument, there are regions where it cannot track the information and missing data occur. An image of one day Ozone density data tracked by OMI is shown in Fig. 1, implemented from [11]. This kind of image will be used for the reconstruction. The missing data are represented as the

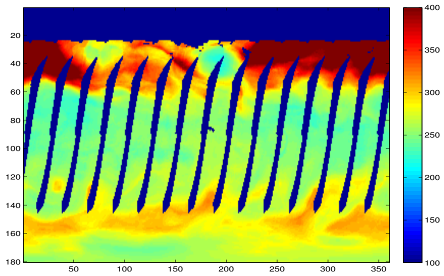

Fig. 1. The available Ozone density data for one day

blue stripes on the image. The reconstruction of missing data on this kind of information is a complex and costly insufficient task to be solved on the instrument hardware. It turns out that it is easier to do it on the software part of the instrument. In this case, CS approach will be used in a specific way. If we assume that the signal representing Ozone data is sparse, we can reconstruct the missing parts of data if the CS reconstruction conditions are satisfied. So in the reconstruction we will consider the blue stripes as unavailable/missing samples and assume that the data are sparse in the two-dimensional Discrete Cosine Transform (2D-DCT) on appropriately defined blocks.

B. CS Problem Formulation

Within the sparse signal processing framework, the Ozone density can be described as a two-dimensional dynamic signal. Consider the original Ozone data as an $M \times N$ dynamic signal $\mathbf{x}_t$ at a frame corresponding to the measurement time t. The signal can be written in vector form as

$$\mathbf{x}_t = [x_t(0,0), ..., x_t(M-1, N-1)]^T. \quad (1)$$

As a transformation domain the two-dimensional DCT will be used. It is calculated as

$$X_t(k,l) = \sum_{m=0}^{M} \sum_{n=0}^{N} x_t(m,n)\varphi(k,l,m,n), \quad (2)$$

where $\varphi(k,l,m,n)$ are two-dimensional DCT basis functions

$$\varphi(k,l,m,n) = c_k c_l \cos\left(\frac{\pi(2m-1)k}{2M}\right) \cos\left(\frac{\pi(2n-1)l}{2N}\right) \quad (3)$$

with $c_{k,l}$ being scaling constants $c_k = 1/\sqrt{M}$ for $k = 0$, $c_k = \sqrt{2/M}$ for $k \neq 0$.

If the basis functions are rearranged into an appropriate matrix $\mathbf{\Phi}$, then for the frame t, the DCT transform and its inverse can be written as

$$\mathbf{X}_t = \mathbf{\Phi}\mathbf{x}_t, \quad \mathbf{x}_t = \mathbf{\Psi}\mathbf{X}_t, \quad (4)$$

where $\mathbf{\Psi}$ is the inverse transform matrix.

Next the assumption that the Ozone data is a sparse signal in the two-dimensional DCT domain will be made. The signal $x_t(m,n)$ is K-sparse in the considered transformation domain if the number of nonzero coefficients K in $X_t(k,l)$ is such that $K << MN$. Sparse signals can be reconstructed from much fewer number of randomly positioned signal samples (that may also be considered as measurements or linear combinations of the sparse transform coefficients). Assume that the signal is known at a set of time positions $\mathbb{N}_{\mathbb{A}} = \{(m_1,n_1),(m_2,n_2),...,(m_{N_A},n_{N_A})\}$. The N_A signal samples or measurements of a linear combination of $X(k,l)$ are denoted as a vector $\mathbf{y}$. Its values are

$$x(m_i,n_i) = \sum_{k=1}^{M} \sum_{l=1}^{N} X(k,l)\psi(m_i,n_i,k,l). \quad (5)$$

They can be written in matrix form

$$\mathbf{y}_t = \mathbf{A}_t\mathbf{X}_t \quad (6)$$

where $\mathbf{y}_t = [x_t(m_1,n_1),...,x_t(m_{N_A},n_{N_A})]^T$and matrix $\mathbf{A}_t$ is an $N_A \times MN$ measurement matrix defined from transformation matrix $\mathbf{\Psi}$, omitting the rows corresponding to the unavailable samples. The general CS task is to recover the signal using the available measurements. In our case it means to recover the unavailable Ozone measurements using the available ones. Since the signal is sparse in the transformation domain, the minimization is done on the sparsity measure function. The basic sparsity measure would be counting of nonzero coefficients in $X_t(k,l)$. This is done by the common ℓ_0-norm. The sparsity measure can be considered as minimization of $\|\mathbf{X}_t\|_0$ subject to $\mathbf{y}_t = \mathbf{A}_t\mathbf{X}_t$. However, the ℓ_0-norm is not convex and cannot be used in efficient optimization algorithms. The ℓ_1-norm, as the closest convex norm, is used instead of the ℓ_0-norm. With this assumption we can reformulate the problem within the CS framework. The available Ozone data at a frame t are denoted by $\mathbf{y}_t$ and the missing data will be the aim of reconstruction using the formulation

$$\min \|\mathbf{X}_t\|_1 \quad \text{subject to} \quad \mathbf{y}_t = \mathbf{A}_t\mathbf{X}_t. \quad (7)$$

Under some conditions the ℓ_1-norm minimization produces the same result as if the ℓ_0-norm were used.

C. Ozone Density as a Dynamic Signal Presentation

For the initial calculation and the graphical presentation the missing measurements in the Ozone data signal will be considered as zero-valued, Fig. 1. Then this signal can be written as

$$x_t^{(0)}(m,n) = \begin{cases} x_t(m,n), & \text{for } (m,n) \in \mathbb{N}_{\mathbb{A}} \\ 0, & \text{elsewhere} \end{cases} \quad (8)$$

with $\mathbb{N}_{\mathbb{A}} = \{(m_1,n_1),(m_2,n_2),...,(m_{N_A},n_{N_A})\}$ being the set of positions of available measurements/samples. The relation between the previous and the present frame is

$$\mathbf{X}_t = \mathbf{X}_{t-1} + \mathbf{q}_t \quad (9)$$

where $\mathbf{q}_t$ represents the dynamic change of the coefficients in the previous frame, [12–14]. The vector $\mathbf{q}_t$ is assumed to be sparse to obtain a sparse signal for $\mathbf{X}_t$. If the vector $\mathbf{q}_t$ has non-zero coefficients at the positions different from the $\mathbf{X}_{t-1}$, then the signal has changed. Otherwise, we will assume that the signal sparsity remains unchanged.

The two main steps in the reconstruction of dynamic signals are prediction and update. In the prediction step, the analysis of the dynamic signal at the frame t is done, by using the key information from previous frames. In the update step, the coefficients at the present frame t are adapted.

III. Reconstruction Algorithm

The gradient algorithm is based on efficient minimisation of the sparsity (7) by varying the unavailable samples (8). This algorithm for one-dimensional case is introduced in [7], [8] and in two-dimensional form in [15], [16]. The dynamic signal reconstruction algorithm is presented in [16]. For an $M \times N$ two-dimensional signal $x_t(m,n)$ at a frame t the sparse signal in the 2D-DCT domain the reconstruction algorithm can be described in six steps.

Step 1: Initialize the signal values set of unavailable samples $(m,n) \notin \mathbb{N}_{\mathbb{A}}$ to zero. The signal at a frame t will then be defined as

$$x_t^{(0)}(m,n) = \begin{cases} x_t(m,n), & \text{for } (m,n) \in \mathbb{N}_{\mathbb{A}} \\ 0, & \text{elsewhere.} \end{cases} \tag{10}$$

In the case of Ozone data, this step is already given in this form. The blue stripes in Fig. 1 are the zero values at the positions of unavailable samples.

Step 2: Add a value of $\pm\Delta$ to an unavailable sample. The new signals can be written as

$$\begin{aligned} x_{t1}(m,n) &= x_t^{(p)}(m,n) + \Delta\delta(m-m_i, n-n_i) \\ x_{t2}(m,n) &= x_t^{(p)}(m,n) - \Delta\delta(m-m_i, n-n_i) \end{aligned} \tag{11}$$

with p being the iteration number. The step Δ is taken to be the maximal absolute value of the available data values.

Step 3: For the available data, the gradient value $g_t(m_i,n_i)$ is zero, which means that they do not change. The gradient value corresponding to an unavailable sample is estimated as a difference of ℓ_1-norms of signals formed in the previous step

$$g_t(m_i,n_i) = \frac{\|\mathbf{X}_{t1}\|_1 - \|\mathbf{X}_{t2}\|_1}{2\Delta} \tag{12}$$

where $\mathbf{X}_{t1} = DCT2\{\mathbf{x}_{t1}\}$, $\mathbf{X}_{t2} = DCT2\{\mathbf{x}_{t2}\}$.

Step 4: The missing values $(m,n) \notin \mathbb{N}_{\mathbb{A}}$ at the present iteration p are updated using the knowledge of the values from the previous iteration $p-1$ as

$$x_t^{(p)}(m_i,n_i) = x_t^{(p-1)}(m_i,n_i) - \mu g_t(m_i,n_i) \tag{13}$$

with a step μ opposite of the gradient. The available signal values are not changed.

Step 5: After a number of iterations, when the reconstructed values are close to the result, they will oscillate around it. This is because of the nature of the gradient of this sparsity measure. Therefore, we should decrease the step size, for example as for $\Delta = \Delta/10$ and $\mu = \mu/10$ and continue the reconstruction. We continue with the process until the desired reconstruction precision is achieved. The reconstruction of the present frame is stopped when the change in two successive iterations is smaller than the desired precision ε

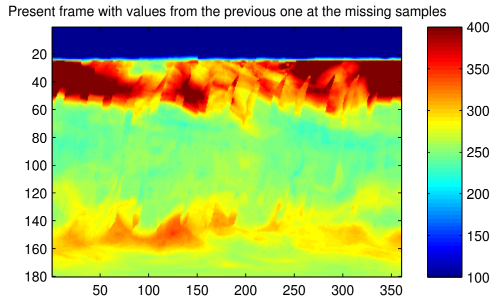

Fig. 2. The data from 26^{th} January with the data reconstructed for 25^{th} January

$$\max_{m,n} \left| x_t^{(p)}(m,n) - x_t^{(p-1)}(m,n) \right| < \varepsilon \tag{14}$$

for $(m,n) \notin \mathbb{N}_{\mathbb{A}}$. The Ozone data is an integer of order 10^3. Therefore, the acceptable relative error between two successive iterations of reconstruction should of 10^{-4} order for a successful reconstruction. Additionally, from the theory of CS we know that the reconstruction of a sparse signal leads to a sparse unique solution. When the desired precision is achieved, we go to Step 6.

Step 6 (Prediction): When a frame is successfully reconstructed, we move on to the next frame. The difference we make in comparison to the first frame is that we do not start from zero values at the unavailable samples. This time, we rather set the values from the previous frame at the positions of the missing samples. Note that the value at the positions of missing samples can be available in the previous frame and vice versa. The initial signal will then be written as

$$x_t^{(0)}(m,n) = \begin{cases} x_t(m,n), & \text{for } (m,n) \in \mathbb{N}_{\mathbb{A}} \\ x_{t-1}(m,n), & \text{elsewhere} \end{cases} \tag{15}$$

where $x_t(m,n)$ are the available data and $x_{t-1}^{(p)}(m,n)$ are the data reconstructed from the previous frame at the positions of the missing (corrupted) data. An example of the signal taking the values from the previous frame is shown Fig. 2.

When the signal in the new frame is defined, we go back and repeat Steps from 2 to 6. When each frame is recovered, the reconstruction is finished. In (15) the simplest prediction form $x_t^{(0)}(m,n) = x_{t-1}(m,n)$ is used for the missing measurements. The prediction can be improved using more advanced methods. For example, applying an L-order adaptive LMS system

$$\begin{aligned} x_t^{(0)}(m,n) &= h_0(t)x_{t-1}(m,n) + ... + h_{L-1}(t)x_{t-L}(m,n) \\ &= \mathbf{h}(t)\mathbf{x}_t(m,n) \\ \mathbf{h}(t) &= \mathbf{h}(t-1) + \mu e(m,n)\mathbf{x}_{t-1}(m,n) \end{aligned} \tag{16}$$

where in prediction setup $e(m,n) = x_t(m,n) - \mathbf{h}(t)\mathbf{x}_t(m,n)$. Additional prediction improvement can be achieved using more advanced adaptive systems or Kalman filters in this step, [13].

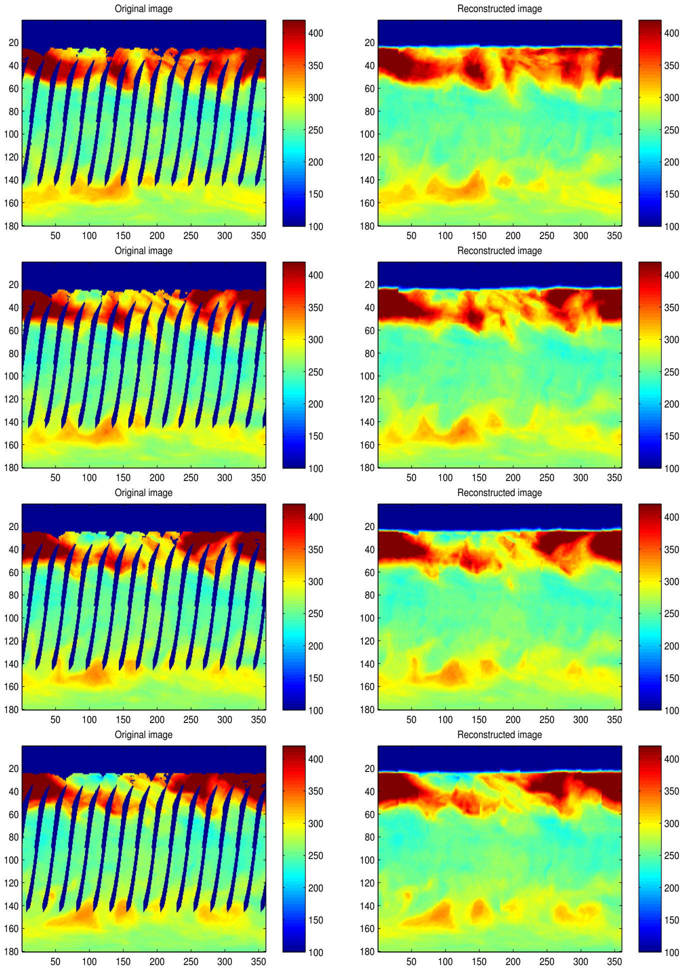

Fig. 3. Corrupted Ozone density data (left), and reconstruction data (right); for days from 25^{th} (top) to 28^{th} (bottom) of January 2016

IV. Numerical Results

We present the reconstruction of the Ozone density data for days from 25^{th} to 28^{th} of January 2016. The data is taken from original OMI documented files, imported from [11]. The data with unavailable measurements are shown in Fig. 3 (left). The reconstructed data are shown in Fig. 3 (right). In our case, the relative error between two successive iterations, for a successful reconstruction, was of order 10^{-4}. A comparison between the Orthogonal Matching Pursuit algorithm, Hierarchical Bayesian-Kalman filter, and the presented gradient-based algorithm is done in [16]. Also, it can be seen that the first frame needs more time for the reconstruction than other frames for the same precision. For the first frame, the algorithm needs approximately 39.35 seconds for the reconstruction, whereas for every other frame (i.e. when the values from the previous frame are used to predict the unavailable values in the next frame) it needs 22.12 seconds on average.

V. Conclusions

The reconstruction of Ozone density data using a gradient-descent compressive sensing algorithm is presented. Because of the hardware nature of the instrument for tracking the Ozone, the data has missing parts as shown. The gradient-descent algorithm can successfully reconstruct this kind of data because of its characteristic to take the missing samples of data as the variables, leaving the ones which are available unchanged. In this case, we implicitly assumed that the data is sparse in the 2D-DCT domain. Then, as we already had the missing samples set to zero, we try to reconstruct them with a desired precision. Since the reconstruction is done in the time domain, other major advantage of the algorithm is that it does not use the signal sparsity in a strict sense. The difference in other frames calculation to the initial one is that we use the values found in the previous frames and continue the reconstruction process based on appropriate prediction. The reconstruction, after the initial frame, is more efficient.

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