FHSS Signal Characterization Based On The Cross-terms Free Time-Frequency Distributions

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Abstract—**The application of the Cohen class time-frequency distributions has been considered for the analysis of signals in wireless communications. Several distributions are considered: spectrogram, Wigner-Ville, S-method, Born-Jordan, Choi-Williams, rectangular and Gaussian kernel based distribution. The possibility of using time-frequency distributions in decomposition of multicomponent wireless signals is examined. The signal separation procedure is based on eigenvalues and eigenvectors decomposition. Time duration and frequency range are estimated after the separation for each signal component. The theory is supported by the experimental results.**

Keywords - Cohen class, eigenvalues, eigenvectors, time-frequency analysis, signal decomposition, wireless communications

I. INTRODUCTION

In the case of the signals with non-stationary spectrum, the Fourier domain analysis fails to provide satisfactory results. Namely, it cannot provide the information about the time instants when spectral components occur. In order to overcome this drawback in the case of the signals with time-varying frequency content, joint time-frequency distributions (TFDs) have been used in practical applications [1]-[6]. A suitable TFD should provide good concentration in the time-frequency domain and high resolution in both, time and frequency. There is no single TFD that can provide appropriate representation for all types of non-stationary signals. The Short-Time Fourier Transform (STFT) and the spectrogram (as its energetic version) are the oldest and the most commonly used tools in time-frequency analysis. However, using the STFT and the spectrogram, it is difficult to achieve good time-frequency resolution in the case of nonlinear signal phase function. In other cases, time-frequency resolution of the spectrogram highly depends of the window width and shape. In order to improve the resolution of the spectrogram, quadratic TFDs have been introduced [1], and later, more complex, higher order distributions [7]-[9]. The commonly used quadratic distributions include the Wigner distribution (WD), distributions from the Cohen class and the S-method. The WD improves time-frequency resolution and it is not window dependent as the STFT, but it has another drawback that limit its applicability. Namely, in the case of the multicomponent signals, WD introduces cross-terms, that appear due to the quadratic nature of the WD. In some situations, the signal terms (auto-terms) cannot be distinguished from the cross-terms, and time-frequency analysis cannot give accurate representation. In order to eliminate cross-terms, the WD is combined with different low-pass kernel functions, resulting in the Cohen class of distributions. Another solution for the cross-terms suppression called the S-method, provides efficient cross-terms free representation with good auto-terms concentration as in the Wigner distribution. When dealing with multicomponent signals, whose components are very close to each other, the S-method might produce certain cross-terms.

In this paper we consider the Frequency Hopped Spread Spectrum (FHSS) communication signals with very close components, aiming to separate the components and to determine their time duration and frequency range. The components separation method is based on the combination of eigenvalue decomposition and time-frequency distributions. In order to avoid cross-terms, the distributions from the Cohen class are applied and compared to the spectrogram, the WD and the S-method. The analysis includes different types of kernels that should provide cross-terms suppression and at the same time good concentration of auto-terms. The optimal choice of kernel function should provide accurate instantaneous frequency (IF) estimation of signal components. The mean square error (MSE) is measured between the true and the estimated IF. The optimal kernel provides minimal MSE of the IF estimation, while preserving signal features (time duration and frequency range of signal components called hops) as well as the time-frequency concentration of each component.

The paper is organized as follows: Section II is theoretical background on the commonly used TFDs. In Section III, the eigenvalue decomposition method, based on the Cohen class of distributions, is described. The experimental results are discussed in Section IV. MSEs and duration measures for different distributions are provided in this section as well. Concluding remarks are given in Section V.

II. THEORETICAL BACKGROUND

The STFT is widely used in the time-frequency analysis. It is, in fact, the Fourier transform of the windowed signal, which can be defined as [1], [3]:

$$STFT(t,\omega) = \int_{-\infty}^{\infty} x(t+\tau)w(\tau)e^{-j\omega\tau}d\tau, \quad (1)$$

where $x(t)$ is signal and $w(\tau)$ is a window function. In the case of multicomponent signals, STFT is equal to the sum of the STFTs of each individual component. STFT resolution in the TF plane is window dependent, which is its main drawback. In order to improve signal resolution in the TF plane, the Wigner distribution is used [1]:

$$WD(t,\omega) = \int_{-\infty}^{\infty} x(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2})e^{-j\omega\tau}d\tau. \quad (2)$$

Although the WD improves concentration in the time-frequency plane, it is not suitable for multicomponent signals. Namely, for multicomponent signals, the cross-terms appear between the two auto-term components, and in some cases, can be misinterpreted as the auto-term. As a suitable solution for the cross-terms, the S-method was introduced. It reduces the presence of cross-terms while keeping good auto-terms concentration. The S-method is defined as [7]:

$$SM(t,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} P(\theta)STFT(t,\omega+\frac{\theta}{2})STFT^*(t,\omega-\frac{\theta}{2})d\theta, \quad (3)$$

where $P(\theta)$ is the frequency window function. Reduction of the cross-terms will depend on the width of $P(\theta)$: the window should not be too wide and the signal components should not be very close to each other.

All of the mentioned distributions are just special cases of the general Cohen class of distributions [1], [10]-[12], which is derived using the different low-pass kernel functions:

$$CD(t,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c(\theta,\tau)A(\theta,\tau)e^{-j\theta t-j\omega\tau}d\tau d\theta, \quad (4)$$

where $c(\theta,\tau)$ is 2D kernel function, and $A(\theta,\tau)$ represents the ambiguity function (AF):

$$A(\theta,\tau) = FT_{t,\omega}\{WD(t,\omega)\} = \int_{-\infty}^{\infty} x(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2})e^{-j\theta t}dt. \quad (5)$$

In the ambiguity domain, the auto-terms are located around the origin, and the cross-terms are dislocated. The ambiguity function is filtered by using the kernel function. However, there is a trade-off between cross-terms reduction and auto-terms concentration.

III. EIGENVALUE DECOMPOSITION APPLIED TO THE COHEN CLASS OF DISTRIBUTIONS

In order to separate components of the multicomponent signal, the eigenvalue decomposition method [13] is used. This method is combined with TFDs. In order to introduce eigenvalue decomposition method, let us first consider inverse form of the WD:

$$x(n+m)x^*(n-m) = \frac{1}{N+1}\sum_{k=-N/2}^{N/2} WD(n,k)e^{j\frac{2\pi}{N+1}2mk}, \quad (6)$$

For the signal that consists of M components, the previous relation becomes:

$$\sum_{i=1}^{M} x_i(n+m)x_i^*(n-m) = \frac{1}{N+1}\sum_{k=-N/2}^{N/2}\sum_{i=1}^{M} WD_i(n,k)e^{j\frac{2\pi}{N+1}2mk}, \quad (7)$$

If the STFTs of the signal components do not overlap in the time-frequency plane, then sum of the WDs on the right side of (7) could be replaced with the cross-terms free distribution from the Cohen class:

$$\sum_{i=1}^{M} x_i(n+m)x_i^*(n-m) = \frac{1}{N+1}\sum_{k=-N/2}^{N/2} CD(n,k)e^{j\frac{4\pi}{N+1}mk}. \quad (8)$$

Left side of (8) is the autocorrelation matrix C to which we apply the decomposition method. Eigenvalue decomposition of the square matrix C could be written as:

$$C = \sum_{i=1}^{N+1}\eta_i v_i(n)v^*_i(n). \quad (9)$$

In relation (9) η_i are eigenvalues and v_i are eigenvectors of matrix C. Eigenvectors correspond to the signal components, while eigenvalues correspond to the energy of components.

IV. EXPERIMENTAL RESULTS

The eigenvalue decomposition procedure is applied on closely-spaced FHSS signal. FHSS [14]-[17] is a modulation technique, used for the transmission of radio signals. This modulation technique is based on changing the carrier frequency from one value to another and finds application in the Bluetooth systems. Bluetooth is a communications protocol that uses the ISM (Industrial, Scientific and Medical) frequency band from 2.4 GHz to 2.4835 GHz. There are several standards that share this frequency band. Specific techniques are developing, in order to identify the types of signals in the ISM frequency band. Some of them use TF distributions to characterize the signal. In order to characterize the signal, its features have to be determined. The features are extracted after the separation of the signal components.

Fig.1 shows different TF representations of the FHSS signal. The signal consists of four closely spaced components and is defined as:

$$x(t)=e^{-j4\pi t(1:N/4)}+e^{j8\pi t(N/4:N/2)}+ \\ +e^{-j14\pi t(N/2:3N/4)}+e^{-j5\pi t(3N/4:N)}, \quad (10)$$

where $t=-1:1/128:1$, and N is signal length. The spectrogram (Fig. 1a) does not provide good TF resolution of the signal. In order to improve TF resolution, the S-method is considered. It can be seen that S-method improves resolution, but introduces cross-terms between two neighboring components (Fig. 1b). It is still possible to differ between auto and cross-terms, but signal components could not be separated. The WD (Fig. 1c) introduces large number of cross-terms, and make differentiation between auto and cross terms impossible.

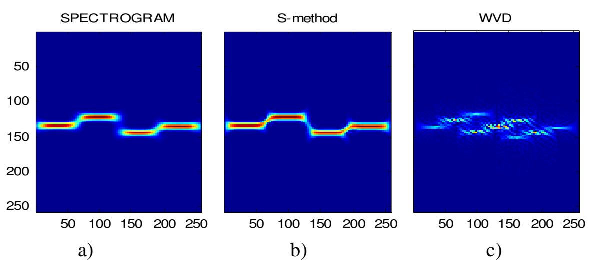

Figure 1. a) Spectrogram, b) S-method, c) Wigner distribution

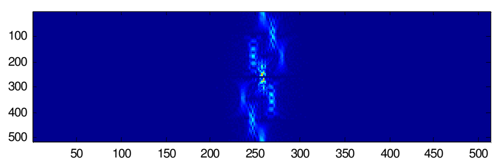

Figure 2. Ambiguity function of the observed FHSS signal

The decomposition procedure could be applied only if the signal components do not overlap in the TF plane. The TF representation must be cross-terms free if we want to separate only the auto-terms. In order to reduce the cross-terms and to keep the best possible resolution of the signal, different distributions from the Cohen class are observed [12]. They are based on the AF filtering. The AF of the considered signal is presented in Fig. 2. TF distributions of the observed FHSS signal, obtained by using different kernels, are shown in Fig. 3. It is shown that, by adapting the Gaussian kernel, it is possible to completely remove the cross-terms and to keep other signal features, like good resolution in the TF plane, frequency range and time duration of the each component. Components, separated from the distribution based on Gaussian kernel, are shown in Fig. 4. Table I shows time duration and the frequency range of the each separated component, estimated by using different kernels. The rectangular kernel gives the worst results. BJ, CW and Gaussian kernel give the similar results, except Gaussian kernel keeps the rectangular shape of the signal components in the TF plane, which is important feature for signal classification.

Fig. 5 shows true IF of the signal (solid line), and IF estimated from the Cohen class (dotted line), for different types of kernel. Also, in Table II we provide MSEs of the IF estimation. The smallest MSE is obtained for the distribution from Fig. 3h, obtained with Gaussian kernel, which is the closest to the true IF of the considered signal.

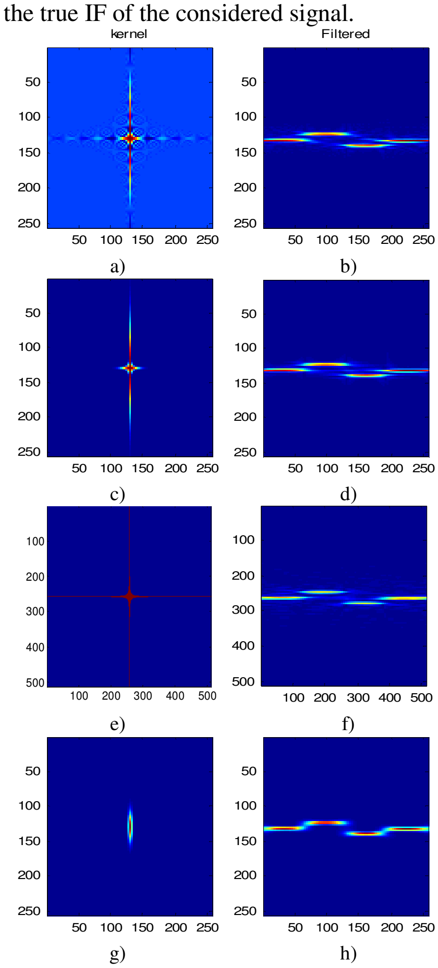

Figure 3. Born-Jordan, Choi-Williams, rectangular, Gaussian kernel (left column, a, c, e, g) and time-frequency representation obtained by using these kernels (right column, b, d, f, h)

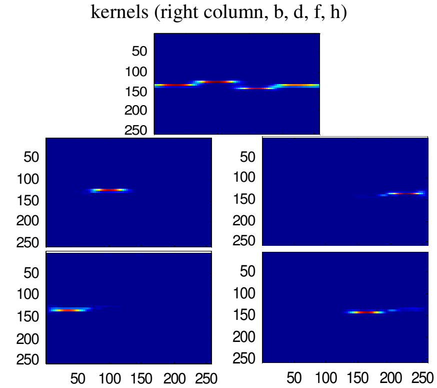

Figure 4. Time-frequency representation obtained by using Gaussian kernel (first row), and separated components (second and third row)

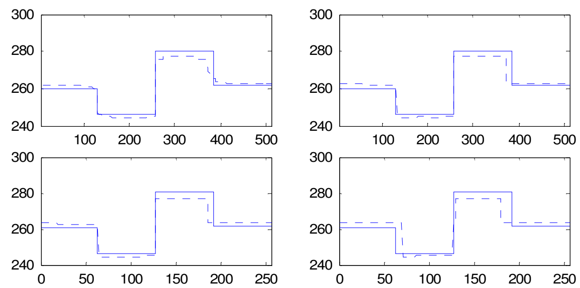

Figure 5. True IF of the signal (solid line) and IF estimated from the separated components (dotted line) for different kernels: Gaussian and Born-Jordan (first row), Choi-Williams and rectangular (second row)

TABLE I: TIME AND FREQUENCY DURATION OF THE EACH SIGNAL COMPONENT

Kernel	Duration							
	Comp. 1		Comp. 2		Comp. 3		Comp. 4	
	t	ω	t	ω	t	ω	t	ω
Born-Jordan	69	12	63	14	62	12	64	11
Choi-Williams	68	11	62	16	62	12	63	11
Rectangular	224	17	55	14	53	14	212	14
Gaussian	56	14	62	16	58	12	69	14

TABLE II: MSE BETWEEN IF ESTIMATED FROM THE COHEN CLASS DISTRIBUTION AND TRUE IF OF THE SIGNAL

Kernel	Mean Square Error
Born Jordan	15.5756
Choi Williams	16.1294
Rectangular	31.2288
Gaussian	10.4912

V. CONCLUSION

The procedure for decomposition of FHSS signals is described in the paper. The decomposition procedure includes the calculation of eigenvalues and eigenvectors from the properly formed autocorrelation matrix. FHSS signals could be very closely frequency spaced, or there could be other signals presented in the ISM frequency band. In the case of the multicomponent signals with closely-spaced components, the decomposition procedure may fail if the components are not properly spaced (if there are overlappings or cross-terms). In order to overcome this problem, decomposition is combined with distributions from the Cohen class. It has been shown that, by properly adapted kernel shape, the Cohen class TFDs based on Gaussian kernel can provide reduction of the cross-terms, providing at the same time the best concentrated components in the TF plane. The results are verified by measuring the MSE of IF estimation for signal components.

ACKNOWLEDGMENT

The work is supported by the Ministry of Science of Montenegro.

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