

# Combined Compressive Sampling and Image Watermarking

Irena Orović and Srdjan Stanković

University of Montenegro, 20000 Podgorica, Montenegro  
*irenao@ac.me*

**Abstract** - A procedure for image watermarking in the presence of compressive sampling is proposed. The randomly chosen measurements from image blocks are used to carry the watermark. The image reconstruction based on a set of watermarked measurements is performed using the total variation minimization algorithm, allowing good quality of recovered image. The watermark can be efficiently detected within the recovered image, but only if we know the random measurement matrix used for image acquisition. This provides secure watermarking scenario. The watermark detection is tested in the examples using various images.

**Keywords** - Image watermarking, Compressive Sampling, TV minimization

## I. INTRODUCTION

According to the Shannon-Nyquist sampling theorem, signal information is preserved only if the signal is sampled at the rate that is at least twice higher than the maximal signal frequency. In the case of signals characterized by high-frequency content or in the case of high resolution images, this will result in large number of samples. Storing and transmission of these signals may be difficult and sometimes not feasible. Also, in some applications such as medical and radar imaging, increasing sampling rate can be very expensive. In the last few years, a method called Compressive Sensing/Sampling (CS) has been developed, as an alternative for signal acquisition [1]-[7]. Under the certain conditions, the signals can be reconstructed from a small number of random measurements, whereas the signal must fulfill certain conditions, such as sparsity. The information about sparse signal is contained in the significantly smaller number of coefficients, compared to the total length of the signal. The reconstruction of CS signals is based on different optimization algorithms. For instance, in image applications, total variation (TV) minimization algorithms are usually applied. When dealing with a small incomplete set of randomly chosen image samples, we can hardly apply standard image processing algorithm. However, we still have to provide some data protection algorithms, such as digital watermarking [8]-[12]. The watermark embedding and detection are usually done in DFT, DCT, DWT domain or in time-frequency domain using time-varying mask [13]. In this paper we deal with watermarking of compressive sampled images based on sparse DFT image representation. Further, we analyze the possibility to reconstruct image from such a small set of data, in order to

provide successful watermark detection after image reconstruction.

The paper is organized as follows. After the Introduction, in Section II we present the basic theoretical concepts belonging to the compressive sensing approach. Section III is devoted to TV reconstruction methods. The combined compressive sensing and watermarking procedure is proposed in Section IV. The results of image reconstruction and watermark detection are presented experimentally in Section V. The concluding remarks are given in Section VI.

## II. COMPRESSIVE SENSING

Nowadays, the CS has been used in various applications involving one-dimensional and two-dimensional signals [1]-[7], that appear in antenna arrays, indoor and SAR imaging, communications, remote sensing, multimedia applications, etc. CS deals with the signals which are sparse in a certain transform domain. It means that the signals have concise representations when expressed in the proper basis. The sparse signal to be recovered can be sparse in its own domain or in some of the transform domains that might be based on DFT, DCT, or using any other orthogonal basis expansion. Sparse approximation in orthogonal bases is the essence of many efficient compression and denoising algorithms. In general, a signal which is  $K$  sparse in a specific domain can be completely characterized by  $M$  measurements ( $M > K$ ) with  $M \ll N$ , where  $N$  is the number of samples imposed by the Shannon-Nyquist theorem.

Let us consider a discrete-time signal  $x$  of length  $N$ . Any signal can be represented in terms of basis vectors as follows:

$$x = \sum_{i=1}^N s_i \psi_i = \psi s, \quad (1)$$

where  $s_i$  represents the transform domain coefficient,  $\psi_i$  is a basis vector,  $\psi$  denotes  $N \times N$  transform matrix whose columns are basis vectors. If only  $K$  transform coefficients from  $s$  have non-zero values, we can say that  $x$  is  $K$ -sparse in transform domain defined by  $\psi$ .

Signal measurements are acquired from the domain where signal have "dense" representation, where  $M \ll N$  holds. Despite the dimensionality reduction, the information needed to recover the signal is well preserved, if the procedure

satisfies certain conditions. Firstly, the measurement matrix  $\phi$  must be incoherent with the basis matrix  $\psi$ . The coherence between two matrices measures the largest correlation between any two elements of matrices [5]:

$$\mu(\phi, \psi) = \sqrt{N} \max_{k \geq 1, j \leq N} \left| \langle \phi_k, \psi_j \rangle \right|, \quad (2)$$

where  $N$  is the signal length,  $\phi_k$  and  $\psi_j$  are row vector and column vector of the  $\phi$  and  $\psi$  matrices, respectively. The coherence has values in the range:

$$1 \leq \mu(\phi, \psi) \leq \sqrt{N}, \quad (3)$$

and it increases as the elements of two matrices are more correlated. Lower coherence between  $\phi$  and  $\psi$  leads to a smaller number of measurements required to recover the entire signal. This number can be estimated as follows:

$$M \geq cK \log(N / K), \quad (4)$$

where  $c$  is a constant. The case of interest is when the number of the required measurements is much smaller than the length of the signal. If the measurement vector is denoted as  $y$ , then the signal measuring procedure can be defined using the measurement matrix  $\phi$  as follows:

$$y_{M \times 1} = \phi_{M \times N} x_{N \times 1}, \quad (5)$$

where  $\phi$  is measurement matrix. From (1) and (5) follows:

$$y = \phi x = \phi \psi s = \theta s. \quad (6)$$

The system of equations defined by (6) consists of  $M$  equations with  $N$  unknowns. Therefore, the system is undetermined ( $M \ll N$ ) and has infinite number of solutions. In order to obtain optimal solution, various minimization algorithms have been used. Each of them is based on finding the sparsest solution. It is shown that optimal results are provided by using optimization techniques based on the  $l_1$  - minimization:

$$\hat{s} = \min \|s\|_{l_1} \quad \text{subject to } y = \theta s, \quad (7)$$

where  $\hat{s}$  is a solution of the minimization problem, whereas the  $l_1$  - norm of vector  $s$  is defined by:

$$\|s\|_{l_1} = \sum_{i=1}^N |s_i|. \quad (8)$$

The problems in CS can be recast as linear programs or they can be recast as second-order cone programs. Generally, the linear programming problems are usually solved using the basis pursuit primal-dual methods, while the second-order

cone programs are solved using a generic log-barrier algorithm.

### III. TV MINIMIZATION ALGORITHM

If the underlying signal is a 2D image, an alternative recovery model is that the gradient is sparse. One of the approaches used in various image processing applications is based on the total-variation of an image [7]. An example of using the TV method is in denoising and restoring of noisy images. If  $x_n = x_0 + e$  is a ‘‘noisy’’ observation of  $x_0$ , we can restore  $x_0$  by solving the following minimization problem [7]:

$$\min_x TV(x) \quad \text{s.t. } \|x_n - x\|_{\ell_2}^2 < \varepsilon^2, \quad (9)$$

where  $\varepsilon = \|e\|_{\ell_2}^2$  should hold and TV denotes the total-variation. The total-variation of  $x$  represents the sum of the gradient magnitudes at each point and can be approximated as:

$$TV(x) = \sum_{i,j} \|D_{i,j} x\|_{\ell_2}, \quad (10)$$

where the gradient is defined as:

$$D_{i,j} x = \begin{bmatrix} x(i+1, j) - x(i, j) \\ x(i, j+1) - x(i, j) \end{bmatrix}. \quad (11)$$

The equality constrained TV minimization problem for a measurements vector  $y$  and a sparse vector of transform domain coefficients  $s$  can be defined as follows:

$$\min_s TV(s) \quad \text{s.t. } y = \theta s. \quad (12)$$

The TV based denoising methods tend to remove the noise while retaining the details and edges in an image. Thus, in the presence of noise we may write:

$$\min_s TV(s) \quad \text{s.t. } \|\theta s - y\|_{\ell_2}^2 < \varepsilon^2. \quad (13)$$

It can be recast as the second-order cone problem:

$$\min_{t,s} \sum_{i,j} t_{ij} \quad \text{s.t. } \|D_{ij} s\|_2 \leq t_{ij} \\ y = \theta s, \quad (14)$$

which can be solved using the second-order cone programming based on the log-barrier method.

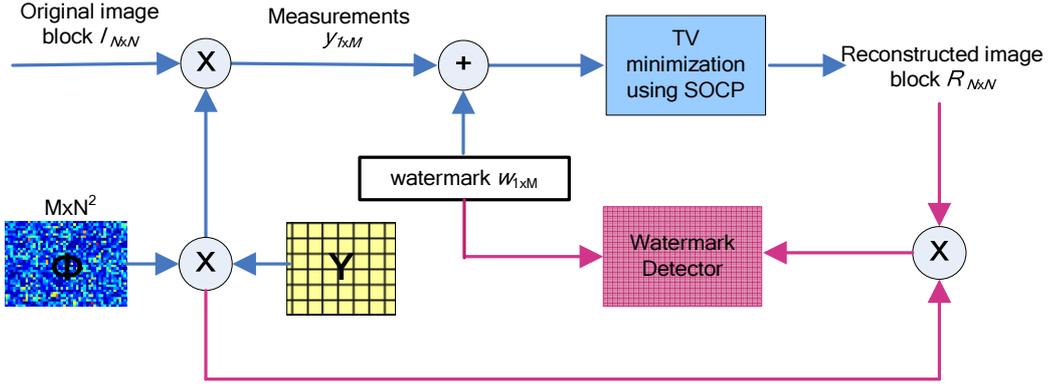


Figure 1. A scheme of CS watermarking procedure

#### IV. CS WATERMARKING PROCEDURE

In the applications that require compressive sampled images either due to the storage or transmission facility, the watermarking cannot be performed in the standard manner and the scenario should be adapted to the compressive sensing principles. In these circumstances, the watermark embedding can be done only within the set of measurements, since these are the available samples. The block scheme of the watermarking system is given in Fig. 1.

The image is firstly divided into blocks and the measurements are selected from each block individually. The samples are taken from the spatial domain where the signal is dense. The DFT coefficients are considered for sparse representation and reconstruction using TV minimization algorithm. Thus, we have:

$$\left. \begin{aligned} y_k &= \phi_k I_k \\ I_k &= \psi F_k \end{aligned} \right\} \Rightarrow y_k = \phi_k \psi F_k, \quad (15)$$

where  $I_k$  represents the  $k$ -th image block of size  $N \times N$ ,  $y_k$  is a vector of measurements for the  $k$ -th block, whereas  $F_k$  is the vector of  $k$ -th block Fourier coefficients. The watermark embedding is done as follows:

$$y_{kw} = y_k + \alpha w_k, \quad (16)$$

where the parameter  $\alpha$  controls the watermark strength. Watermark is created as a pseudo-random sequence, and the watermark length is determined by the number of measurements  $M$ . At the decoder side, the watermarked measurements are used for image reconstruction based on the TV minimization algorithm:

$$\begin{aligned} \min_F TV(F_k) \quad s.t. \quad y_k &= \psi \phi F_k \\ R_k &= \psi F_k \end{aligned} \quad (17)$$

where  $R_k$  is the reconstructed image.

#### A. Watermark Detection

A blind watermark detection is done using the standard correlation detector, applied to the watermarked coefficients:

$$Det(w) = \sum_i y_{iw} w_i. \quad (18)$$

Hence, during the watermark detection procedure we need the random measurement matrix  $\phi_k$ , in order to select the watermarked measurements. Otherwise, the watermark cannot be detected, and  $\phi_k$  actually provide a high level of security.

For any wrong trial (wrong key),  $Det(w) > Det(wrong)$  should hold. The detectability index, from signal detection theory, has been used to evaluate detection performance. Thus, the detection is performed for all right keys and wrong trials. Then the mean values of detector responses are calculated:  $\bar{D}(w)$  for watermarks (right keys) and  $\bar{D}(wrong)$  for wrong trials. The standard deviations of detector responses ( $\sigma_w^2$  for watermark (right key) and  $\sigma_{wrong}^2$  for wrong trials) are calculated, as well. Considering these parameters, the measure of detection quality is obtained as [14], [15]:

$$R = \frac{\bar{D}(w) - \bar{D}(wrong)}{\sqrt{\sigma_w^2 + \sigma_{wrong}^2}}. \quad (19)$$

The measure  $R$  is further used to calculate the probability of detection error as follows:

$$P_{err}(R) = \frac{1}{4} \operatorname{erfc}\left(\frac{R}{2}\right) - \frac{1}{4} \operatorname{erfc}\left(-\frac{R}{2}\right) + \frac{1}{2}. \quad (20)$$

#### V. EXPERIMENTAL RESULTS

The advantages of the proposed watermarking procedure are shown in the sequel. The procedure is applied to the image blocks of size  $16 \times 16$ . The measurements vector in each block consists of 40% of randomly chosen image samples. For the sake of simplicity, but without loss of generality, we have assumed that  $\phi_k = \phi$ , i.e. that the random positions for image

sensing are the same in each image block. The obtained PSNR after compressive sensing reconstruction of watermarked measurements is approximately 35dB, indicating good image quality, especially when we have in mind that the pixel acquisition process actually provided only 40% of random pixels. The original images and reconstructed CS watermarked images are shown in Fig 2.

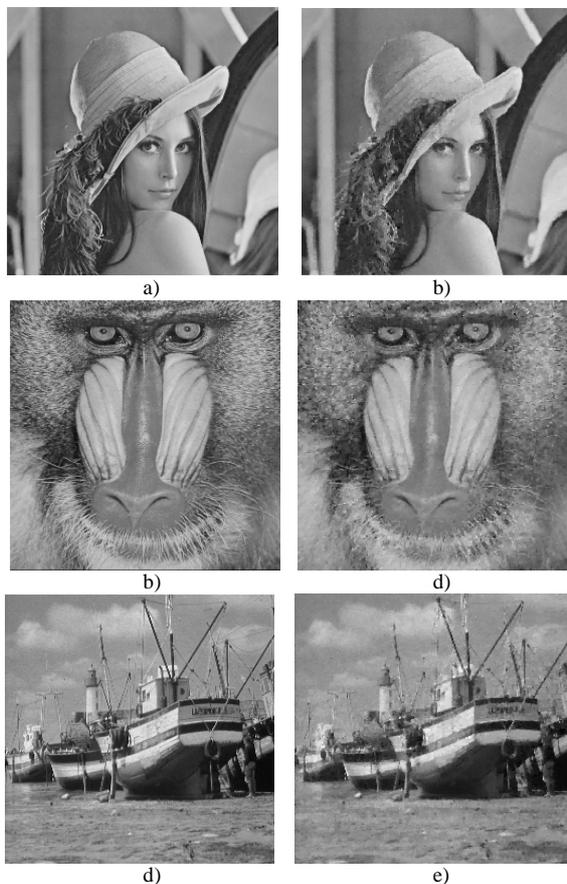


Figure 2. a) Original image Lena, b) CS reconstructed and watermarked Lena, c) Original image Baboon, d) CS reconstructed and watermarked Baboon, e) Original image Boat, f) CS reconstructed and watermarked Boat

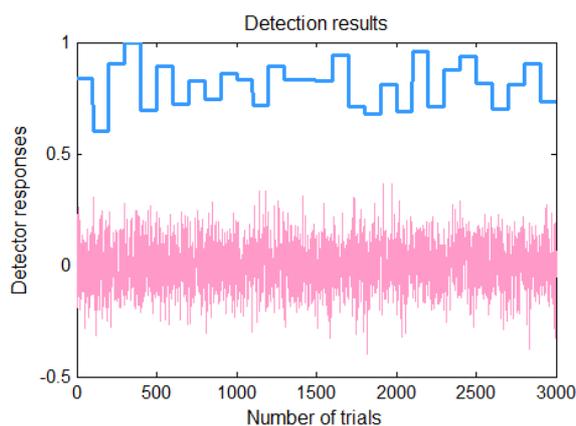


Figure 3. Results of watermark detection for 30 right keys and 3000 wrong trials

The results of watermark detection are given in Fig. 3, for 30 right keys (watermarks), whereas for each watermark we created additional 100 wrong trials to illustrate the efficiency of detection. The achieved measure of detection quality is 6.8, resulting in very low probability of error  $10^{-6}$ .

## VI. CONCLUSION

An image watermarking procedure for compressed sampled images is proposed. It has been shown that the reduced set of measurements can carry the watermark, while the watermark does not influence image reconstruction afterwards. The high quality reconstruction is achieved using the TV minimization algorithm. The watermark detection assumes selection of watermarked coefficients from the reconstructed image according to the random measurement matrix. The detection is demonstrated for a number of watermarks and random trials.

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## REFERENCES

- [1] M. Fornasier, and H. Rauhut, "Compressive sensing", (Chapter in Part 2 of the *Handbook of Mathematical Methods in Imaging*, Springer, 2011
- [2] S. Stanković, I. Orović, E. Sejdić, "Compressive sensing," (Chapter in *Multimedia Signals and Systems*), Springer 2012.
- [3] Y. Yoon, and M. Amin, "Compressed sensing technique for high-resolution radar imaging," *Proc. SPIE*, 6968, 6968A-6968A-10, 2008.
- [4] A.C. Gurbuz, J.H. McClellan, W. R. Scott, Jr., "A Compressive Sensing Data Acquisition and Imaging Method for Stepped Frequency GPRs," *IEEE Trans. Geoscience and Remote Sensing*, vol. 57. no. 7, 2009, pp. 2640–2650
- [5] D. L Donoho, "Compressed sensing," *IEEE Trans. Info. Theory*, vol. 52, no. 4, 2006, pp. 1289–1306
- [6] Chen, S., Donoho, D., and Saunders, M., "Atomic decomposition by Basis Pursuit," *SIAM J. Sci. Comput.*, vol. 20, no.1, 1999, pp. 33–61.
- [7] E.J. Candes, J. Romberg, T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. on Information Theory*, vol.52, no.2, pp. 489-509, 2006
- [8] E. Muharemagić, B. Furht, "Survey of watermarking techniques and applications," CH. in *Multimedia Watermarking Techniques and Applications*, B. Furht and D. Kirovski, editor, Auerbach Publication, pp. 91-130, 2006
- [9] I. J. Cox, M. L. Miller, J. A. Bloom, "Digital Watermarking", Academic Press, 2002.
- [10] S. Stankovic, I. Orovic, N. Zaric, "Robust watermarking procedure based on JPEG-DCT image compression," *Journal of Electronic Imaging*, vol. 17, no. 4, 043001, 2008
- [11] M. Barni, F. Bartolini, "Watermarking Systems Engineering", New York: Marcel Dekker, Inc. 2004.
- [12] S. Stankovic, I. Djurovic, R. Herpers, LJ. Stankovic, "An approach to the optimal watermark detection," *AEUE International Journal of Electronics and Communications*, vol. 57, no.5, 2003, pp. 355-357
- [13] S. Stankovic, "About Time-Variant Filtering of Speech Signals with Time-Frequency Distributions for Hands-Free Telephone Systems," *Signal Processing*, vol.80, no.9, 2000.
- [14] S. Stankovic, I. Orovic, N. Zaric, "An Application of Multidimensional Time-Frequency Analysis as a base for the Unified Watermarking Approach," *IEEE Trans. on Image Processing*, vol. 1, no.3, 2010, pp. 736-745
- [15] I. Orovic, M. Orlandic, S. Stankovic, "An Image Watermarking Based on the pdf Modeling and Quantization Effects in the Wavelet Domain," *Multimedia Tools and Applications*, 2012, pp. 1-17.