A Novel Clipping Technique for Filtering FM Signals Embedded in Intensive Noise

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Abstract— **A novel adaptive clipping technique for filtering a constant amplitude frequency modulated (FM) signal embedded in non-*Gaussian* noise is proposed. It is based on analysis and processing of the estimate of probability density function of a FM signal realization. As a result, modifications of two robust estimators of FM signal amplitude are proposed. As shown, these estimators can be used for *Gaussian* and non-*Gaussian* heavy-tail environments. The proposed clipping technique can exploit one or another obtained robust estimate of the signal amplitude for addaptive setting a threshold. Analysis of signal estimate accuracy for different noise environments is carried out. Comparative analysis of the obtained methods and known approaches based on scanning window nonlinear filtering and optimal robust L-DFT form is performed. It is demonstrated that the usage of clipping based technique leads to the considerable improvement of the FM signal filtering efficiency in comparison to the aforementioned known approaches for different noise environments and for a wide range of input SNR values.**

I. Introduction

It is a well-known fact that the performance of communication, radar, navigation and other systems considerably depends both on the type and characteristics of signals used and noise statistical characteristics [1]-[4]. Because of this, frequency (FM) or phase (PM) modulated and manipulated signals have found wide application in systems mentioned above. The theory of FM and PM signal processing for *Gaussian* noise environment and quite large input SNR $\gg 1$ was developed several decades ago [5].

Since then, considerable attention has been paid to analysis of more complicated and practical situations. In particular, input SNR $\gg 1$ is often not so large, it can be of order 1...10 or even less. Besides, in many practical applications noise cannot be considered *Gaussian* [4], [6]. It has been demonstrated that more heavy-tailed distributions like α-stable and other ones occur to be more adequate for simulating properties of real life noise [4], [6]-[10]. Moreover, commonly reliable a priori information on noise statistics is absent or restricted. Then, the task to remove noise becomes more important (in comparison to the case of input SNR $\gg 1$ and *Gaussian* noise) but more complex.

Assume that we have the following generalized knowledge about signal and noise characteristics. An additive noise has a symmetric probability density function (pdf) with respect to the location parameter (usually zero). Besides, this noise can possess heavy tails (coefficient of kurtosis in this case is usually $\gg 0$) [6]. At the same time, exact intensity (variance) and pdf of noise is unknown. Such assumptions and practical situations are typical for modern radar, communication and computer systems and networks [3], [8]. Concerning signal component, we suppose that within observation interval it has a constant amplitude and, at least, tens or hundreds of oscillation periods.

There are various techniques for filtering of such additive mixture of signal and noise. Among them one can mention a scanning window nonlinear filtering (*SWNF*-method) [11] and robust DFT [12], [13] based approaches. However, alongside with observed positive effects, these methods possess some peculiarities and drawbacks. The optimal or reasonable selection of the window aperture size is one of the main problems of the *SWNF* approach. As a

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result, it is necessary to know the signal spectrum in advance for proper choice of *SWNF* type and parameters.

In turn, robust DFT-methods [12] might have another drawback. In case of their application to processing the signals of relatively large number of samples (several hundreds or more) a large amount of calculations and large computation time are required. For example, in case of the method based on the optimal L-DFT forms it is necessary to compute spectrum estimate for all considered values of trimming parameter in order to obtain its optimal value [13]. Besides, the technique based on the robust forms of the DFT could introduce spectral distortions that can deteriorate accuracy [12].

Therefore, the design of new filtering methods able to get around the shortcomings of aforementioned techniques in the considered applications is a crucial task. It is necessary that the methods under design should be robust in wide sense [14]. This means that these methods should be able to perform well enough in cases of *Gaussian* and heavy-tail noises as well as to operate appropriately for a wide range of input SNRs under a limited a priori information concerning what is a real pdf of noise and what is input SNR.

In this paper we propose two novel noise suppression methods based on clipping algorithm. Note that clipping effects observed in processing of FM signals embedded in non-*Gaussian* noise and/or interference have been thoroughly studied in papers of D. Middleton and A. Spaulding [4], [9], [10]. In particular, they analyzed the influence of clipping on output signal statistics. However, a distinctive difference of our technique lies on the fact that we deal with robust estimation of the FM signal amplitude that can be carried out digitally and, then, with adaptive setting the corresponding threshold. The main problem in our technique is to design an accurate and robust algorithm for estimation of the amplitude of the FM signal. For this purpose, modifications of two robust estimators are developed.

The first technique is based on the inter-quantile differences (*PEAK*-estimator). This estimator is originally proposed for blind estimation of noise variance in [15]. However, the images are generally low-pass signals and we need to make several modifications of this technique for estimating amplitudes of signals corrupted by intensive noise. For this purpose we have studied pdf of the modulated signal amplitude for various types of noise and we have analyzed inter-quantile differences for typical noise environments. Based on this analysis, we adjusted correcting factor of the estimator. The second estimator is based on the median of absolute deviations of processed signal realization (*MMAD*-estimator). This is modification of the approach from [11]. Again we need some adjustment of this technique in order to be able to perform accurate estimation for FM signals. It is shown that the designed methods can be used for different noise environments (*Gaussian* and heavy-tailed noise). The results are compared to the nonlinear filtering methods (*SWNF* and robust DFT). It is demonstrated that our methods exhibit better performance.

The paper is organized as follows. The analysis of a realization of noisy FM signal is carried out in Section 2. The problem of amplitude estimation that appears in the case of noise presence is also highlighted in this Section. The proposed modifications of robust estimators are presented in Section 3. Simulation results with comparative analysis of the achieved accuracy for the proposed amplitude estimators are given in Section 4. The application of designed and known methods to FM signal filtering is considered in Section 5 with example that demonstrates the denoising efficiency of the *PEAK*-method.

II. Sample distribution analysis for FM signal noisy realization

The FM and PM signals used in communication systems and various classes of radar systems (e.g., in radio altimeters, Doppler velocity measuring devices [2], etc.) can be written as [1]

$$s(t) = A\sin[\psi(t)] \quad (1)$$

where $\psi(t)$ denotes the oscillation phase; A is the carrier wave amplitude. In the case of PM signal, $\psi(t) = \omega_0 t + k s_M(t)$; for FM signal

- $\psi(t) = \omega_0 t + k \int_0^t s_M(t')dt' + \varphi_0$, where φ_0 denotes arbitrary initial phase, $s_M(t)$ is the modulating signal and ω_0 denotes the carrier wave frequency. Below we assume that signal has several oscillations (at least five) within observation interval $t \in [0;\ T_S]$.

A sampled realization of FM signal $s(n\Delta t)$ corrupted by noise $\nu(n\Delta t)$ can be represented in the following way [2]:

$$z(n\Delta t) = s(n\Delta t) + \nu(n\Delta t) \tag{2}$$

where $\Delta t = 1/F_B$ is the sampling rate, F_B denotes a sampling frequency, $n \in [1; N]$ is a sample index and $N = T_S/\Delta t$ denotes the number of samples in the observed interval.

Due to noise, the values of $z(n\Delta t)$ can be outside the interval $[-A;\ A]$. Suppose now that one a priori knows A or has its accurate estimate $\hat{A}$. Then, the following simple clipping algorithm can be applied for noise suppression

$$z_{FILT}(n\Delta t) = \begin{cases} \widehat{\xi}, & \text{if } z(n\Delta t) > \widehat{\xi} \\ -\widehat{\xi}, & \text{if } z(n\Delta t) < -\widehat{\xi} \\ z(n\Delta t), & \text{otherwise,} \end{cases} \tag{3}$$

where $\hat{\xi}$ is a threshold value which is supposed to be close enough to the true magnitude of FM signal. For $\hat{\xi} = A$ by means of (3) we change only the values outside $z(n\Delta t) \in [-A;\ A]$, i.e., we perform denoising (clipping) for only some percentage of samples.

Note that, in general, clipping is used in radio electronics rather widely. In particular, clipping is applied in analog amplifiers [16] preceding frequency and phase detectors [1]. The topic of the influence of the distortions caused by the nonlinear elements of the FM receiver where one of the elements is the clipping device has been investigated by D. Middleton [4]. However, the purpose and degree of signal clipping for those applications is different than for considered application. First of all, in case of proper selection of $\hat{\xi}$ in (3) we intend to introduce minimal nonlinear distortions in denoised signal whereas for the task described above [16], pre-distortions are not so crucial and they are desirable sometimes. Second, the proposed clipping technique is focused on the reduction of the distortions caused by some types of noise, e.g., *Gaussian* noise, *Laplacian*, and do not consider the interferences which can arise from other communication channels. Third, the proposed approach can be realized only by the digital signal processing methods unlike the approaches implemented in analog amplifiers or FM receivers [9], [10], [16].

For proposed technique (3), the basic task is to obtain appropriately accurate estimates $\hat{A}$ and setting $\hat{\xi}$ approximately equal to $\hat{A}$. In order to do this in proper manner we consider statistics of the sample of $z(n\Delta t)$, $n \in [1; N]$ in details.

Let us investigate a noiseless harmonic signal with the frequency F and the amplitude A:

$$s(n\Delta t) = A \sin[2\pi F n\Delta t] \tag{4}$$

and obtain the analytic expression for pdf for it.

Since signal (4) is a periodic function with period $T = 1/F$ the pdf can be considered within a period. Then the probability of the signal amplitude between two constants can be written as $P(a < s < b) = (\sin^{-1} b/A - \sin^{-1} a/A)/\pi$. It can be extended as $F(x) = P(-A < s < x) = (\sin^{-1} x/A + \pi/2)/\pi$, by which the pdf is obtained as [17], [18]

$$f_s(x) = \frac{1}{\pi A\sqrt{1-(x/A)^2}}. \tag{5}$$

An example of a histogram of a signal (4) is shown in Figure 1. The simplest procedure for amplitude estimation in this case can be the following

$$\hat{A} = \max_n |s(n\Delta t)|. \tag{6}$$

However, the situation changes for noisy signal. If equation (6) is applied to noisy signal, it produces $\hat{A} > A$ in any case. If noise variance increases, the obtained pdf of the values of the signal (4) occurs to differ more considerably from the pdf estimate in Figure 1. To prove this let us obtain the pdf of realization of signal (4) corrupted by additive noise $\nu(n\Delta t)$ analytically. This task can be done in the following way.

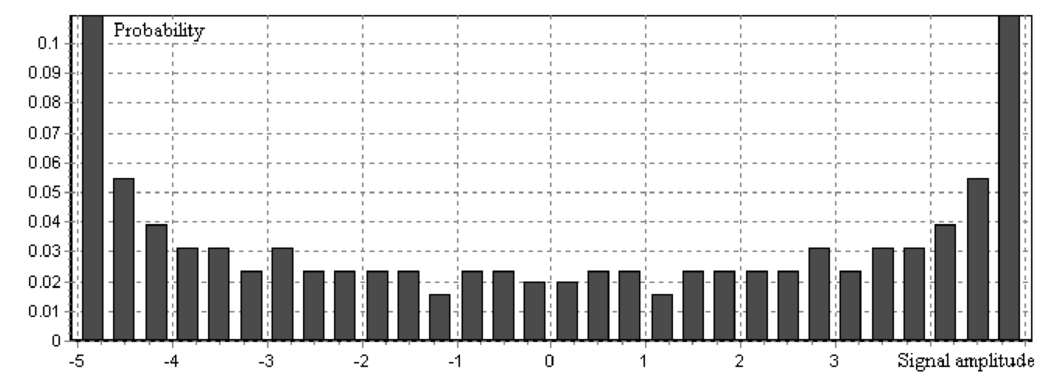

Fig. 1. Histogram of the signal (4) for A=5.

The pdf of the process (2) is defined as [19]:

$$f_z(y) = \int_{-\infty}^{\infty} f_s(x) f_\nu(y-x) dx. \qquad (7)$$

For stationary white *Gaussian* zero-mean noise, $f_\nu(x)$ can be expressed as

$$f_\nu(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right) \qquad (8)$$

where σ denotes the noise standard deviation. Thus, we can rewrite (7) as

$$f_z(y) = \int_{-A}^{A} \frac{1}{\pi A\sqrt{1-(x/A)^2}} \times \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{[y-x]^2}{2\sigma^2}\right) dx. \qquad (9)$$

Carrying the constant items out of the integral, expression (9) becomes

$$f_z(y) = B_1 \int_{-A}^{A} \frac{1}{\sqrt{A^2-x^2}} \times \exp\left(-\frac{[y-x]^2}{2\sigma^2}\right) dx \qquad (10)$$

where $B_1 = \frac{1}{\pi\sigma\sqrt{2\pi}}$. Shape of $f_z(y)$ can be significantly different for various A and σ^2. As examples, Figure 2 represents the pdfs (10) of noisy signal (Figures with indexes *a, c, e*) and pdfs of signal absolute values (Figures with indexes *b, d, f*) for several constant input SNRs defined as

$$SNR_{inp} = P_S/\sigma^2_{inp} \qquad (11)$$

where $P_S = \frac{1}{N}\sum_{n=1}^{N}\left[s(n) - \frac{1}{N}\sum_{i=1}^{N} s(i)\right]^2$ is the signal power, $\sigma^2_{inp} = \frac{1}{N}\sum_{n=1}^{N}(z[n]-s[n])^2 \approx \sigma^2_{add}$ denotes noise variance. For brevity reasons we use notations $s(n) = s(n\Delta t)$, $z(n) = z(n\Delta t)$, $\nu(n) = \nu(n\Delta t)$.

Note that in case of zero-mean symmetric pdf of noise $f_z(y)$ is an even function and, thus, $f_w(y) = 2f_z(y)$ for $0 \le y \le \infty$ and $f_w(y) = 0$ for $y < 0$ where $f_w(y)$ is the pdf of absolute value of noisy signal (5). Another reason for considering $f_w(y)$ will be explained in Section 3. The dependence of the shape of the pdf $f_w(y)$ on input SNR is clearly seen. The pdfs of noise-free FM signal realizations for different instantaneous frequency (IF) laws are practically similar to harmonic signal case (4). The only exception is the fact that depending on the initial FM signal phase and also other parameters (sampling frequency, the length of observation interval, modulating signal, etc.) one can observe the appearance of insignificant pdf skewness. In case of stationary white *Gaussian* corrupted noise with zero-mean noise, the FM signal distribution for input SNR ≈25 has the form similar to that one depicted in Figure 2*a-d*. Slightly differ-

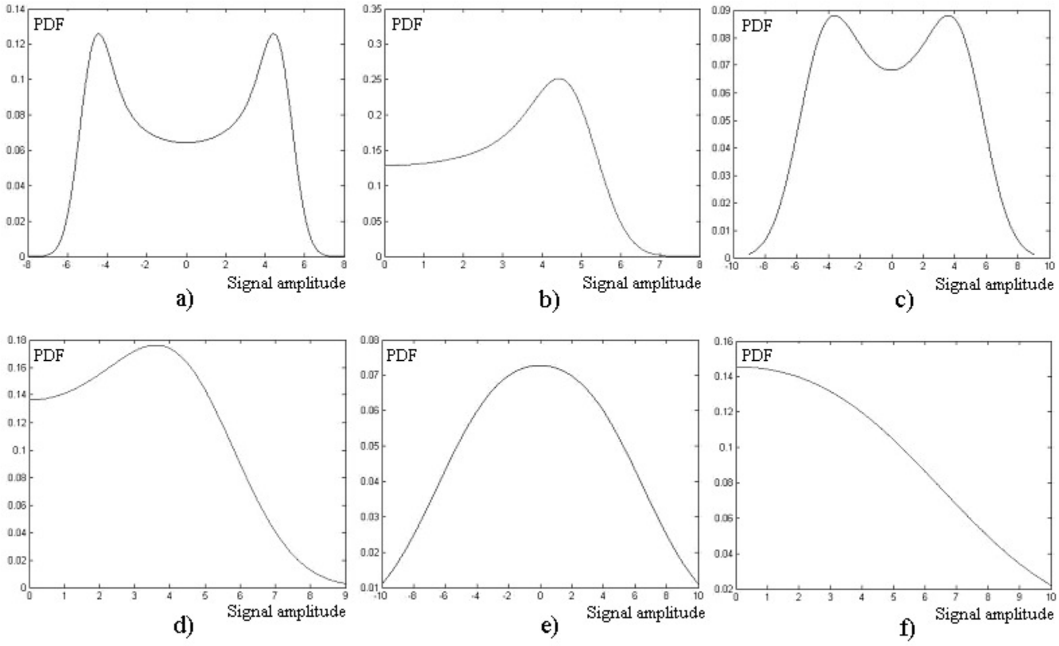

Fig. 2. Pdfs of noisy signal (4) and noisy signal (4) absolute values (for the case of A=5) for input SNR equal to 25 (a, b), 5 (c, d) and 1 (e, f)

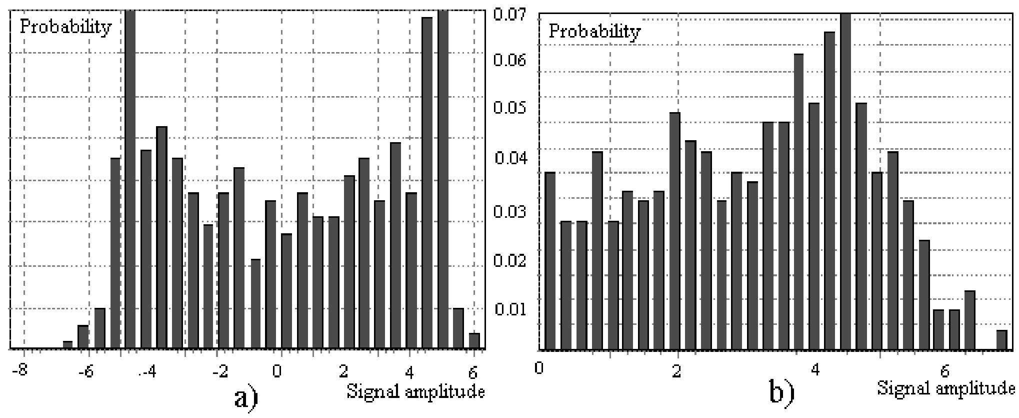

Fig. 3. Histogram of noisy signal (4) (a) and histogram of the absolute values of realization of the signal (4) (b) in case of non-*Gaussian* noise for input SNR≈25.

ent but, in general, similar histograms are observed for both harmonic and FM signals embedded in non-*Gaussian* noise with symmetric and heavy-tail pdf. An example of pdf of the signal (4) embedded in non-*Gaussian* noise is shown in Figure 3. Note that for some heavy-tail distributions analytical expressions for pdf $f_z(y)$ can not be derived whilst for some of them it is possible. For example, for *Laplacian* pdf of noise

$$f_\nu(x) = \frac{1}{\sqrt{2}\sigma} \exp\left(-\frac{\sqrt{2}\,|x|}{\sigma}\right) \tag{12}$$

it is easy to obtain that

$$f_z(y) = B_2 \int_{-A}^{A} \frac{1}{\sqrt{A^2 - x^2}}$$

$$\times \exp\left(-\frac{\sqrt{2}\,(y-x)}{\sigma}\right) dx \qquad (13)$$

where $B_2 = \frac{1}{\pi\sigma\sqrt{2}}$. For the same input SNR, the shapes of $f_z(y)$ in case of *Laplacian* noise are similar to the shapes of $f_z(y)$ presented in Figure 2.

Obviously, amplitude estimation by means of (6) for a limited size samples with distributions described by plots in Figures 2*a*-*d* will lead to incorrect results, i.e., the estimates $\hat{A}$ will be greater than A especially if input SNR is small. Moreover, the estimates $\hat{A}$ have the tendency to become larger if noise has heavier tail.

Therefore, it is necessary to apply other procedures for estimation of FM signal amplitude which would be robust to both *Gaussian* noise and interferences with heavy-tail pdfs. Recall that these estimates have to perform well enough in a wide range of possible input SNRs.

III. Proposed modifications of the robust amplitude estimators

The first proposed way to obtaining an estimate $\hat{A}$ is based on shape analysis of pdf estimate for noisy input signal (Figures 2a-e and 3a). Note that the largest number of sample values is close to A and -A, i.e., the histogram has two main peaks (see Figures 2*a*, *c* and 3*a*). This peculiarity can be exploited for an FM signal amplitude estimation by means of finding a coordinate of the maximum of processed signal realization pdf estimate.

Below we propose modifications of the algorithms based on inter-quantile differences [15] and median absolute deviation (*MAD*) [11] of processed signal realization. Direct application of these algorithms to estimation of the signal amplitude produce biased estimates. To overcome these drawbacks correcting factors are exploited in the modified algorithms. Their values are theoretically evaluated and practically verified for different noise environments.

Now consider each algorithm for amplitude estimation in details. There are several steps for calculation the amplitude estimate based on inter-quantile difference. The first step is to smooth the estimate of pdf and have one maximum for unique defining $\hat{A}$. This can be achieved by processing the absolute values of the realization samples. Here, we take into account that the distributions of magnitudes (2) are practically symmetrical with respect to zero. This statement has been proved by the analysis of the skewness coefficients calculated for various signal-noise setups. Particularly, for a distribution depicted in Figure 3*a* the skewness is not larger than 0.02.

Figure 3*b* shows an example of the histogram obtained after applying transform described above. So, in fact, after such transform we obtain the estimate of distribution of absolute values for the observed signal embedded in noise. Note that this was the reason why in Section 2 pdf $f_w(y)$ is represented and considered in parallel with $f_z(y)$.

It can be seen that the greatest number of the samples has values located close to A. Thus, by means of finding the maximum of the histogram and its corresponding coordinate it is possible to estimate the signal amplitude. The following procedure based on aforementioned idea is proposed for obtaining the amplitude estimate (*PEAK*-estimator):

$$\hat{A}_{PEAK} = K_1 \cdot (X_{(p)} + X_{(q)})/2 \qquad (14)$$

where K_1 is the correcting factor, X_p and X_q denote the p-th and the q-th order statistics of array $|z(n\Delta t)|$.

Parameters p and q are chosen from the relation

$$p - q = \Delta \qquad (15)$$

and their values are calculated from the condition:

$$\min_{p,q}(\left|X_{(p)} - X_{(q)}\right|) \qquad (16)$$

where Δ denotes a priori set constant (15) and $q \in [1; N-\Delta]$. The algorithm described by (14)-(16) is based on the fact that the greatest number of samples has values located near A. Therefore, the distance between order statistics obtained from (15) and (16) will be the smallest near the value A (see for details [15]). This assumption is valid for distributions presented in Figure 2*b* and *d* as well as the pdf histograms given in Figure 3. Thus, finding this minimal distance described by the values

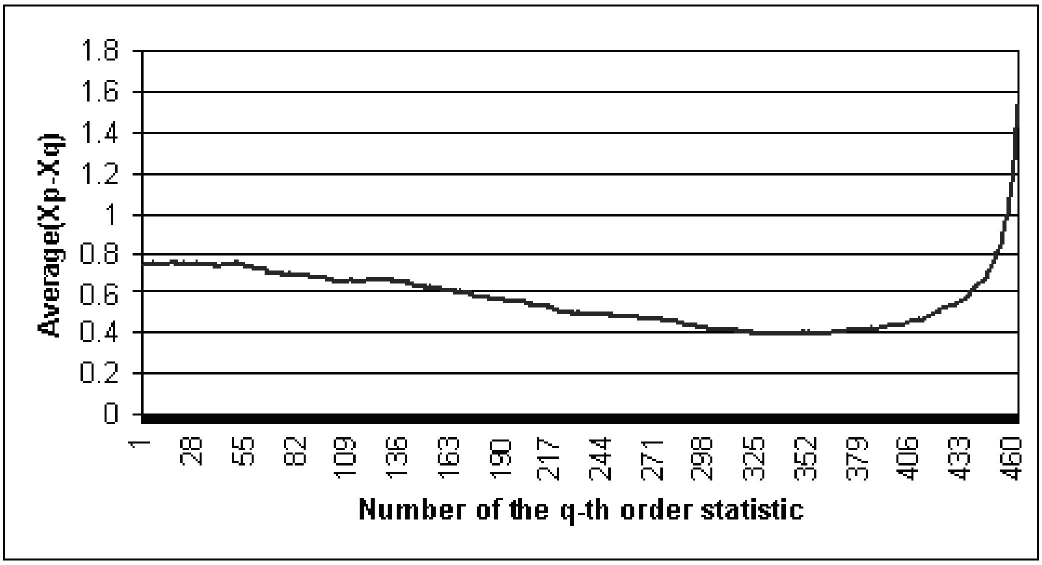

Fig. 4. The dependences of K_1 (a) and K_2 (b) values on input SNR for the noisy realization of signal (4).

of p and q it is possible to obtain the amplitude estimate by means of half-sum of the ordered statistics $X_{(p)}$ and $X_{(q)}$.

At the same time, for the distribution represented in Figure $2f$ the maximal concentration of sample values is observed for neighborhood of zero. Then it can be expected that the clipping technique based on $PEAK$-estimator would not perform well for small input SNRs.

For visual confirmation of this property, Figure 4 shows the dependence of averaged difference $X_{(p)}-X_{(q)}$ on q for the signal (4), $\Delta = 50$, $N = 512$, input SNR=24.79. As seen, the minimum of $X_{(p)} - X_{(q)}$ is observed for $q \approx 350$.

According to (14), it can be expected that the $PEAK$-estimator accuracy depends on parameters K_1 and Δ. Our recommendation on selection of Δ is $\Delta = 0.1N$ (in our simulations $N = 512$ has been used, thus, $\Delta = 51$). Note that the same recommendation on selection of initial value of Δ is given in [15]. Ideally, the correcting factor K_1 should be such that the estimate $\hat{A}_{PEAK}$ is unbiased. This means that the perfect choice of K_1 is $K_1 = A/\left\langle (X_{(p)} + X_{(q)})/2 \right\rangle$ where $\langle \bullet \rangle$ denotes ensemble expectation. However, depending on input SNR and noise pdf, $\left\langle (X_{(p)} + X_{(q)})/2 \right\rangle$ can be different. Thus, the perfect K_1 should depend on input SNR and noise pdf. To prove this, Figure $5a$ represents the dependence of the perfect values of correcting factor K_1 on the input SNR for the case of *Gaussian* noise. It can be seen that the perfect value of K_1 differs for different SNR_{inp}. At the same time, for a wide range of input SNRs (larger than 10) the perfect value of K_1 is rather stable and approximately equal to 1.2. For input SNRs smaller than 10, the perfect value of K_1 increases if input SNR reduces.

Recall that we would like to make our clipping technique simple and robust in wide sense [14]. This means that we would not like to estimate pdf of noise and input SNR. Instead, we prefer to set a fixed value of the correcting factor. Because of this, in our investigations of the $PEAK$-estimator and its statistical characteristics, the value of the correcting factor K_1 is fixed to 1.2.

The second proposed robust FM signal amplitude estimator is based on the calculation of MAD [11] of processed signal realization ($MMAD$-estimator)

$$\hat{A}_{MMAD} = K_2$$

$$\times \mathrm{med}\left\{ \left| X_{(i)} - \mathrm{med}(z_1, z_2, ..., z_N) \right| \right\} \quad (17)$$

where med{...} is the sample median, K_2

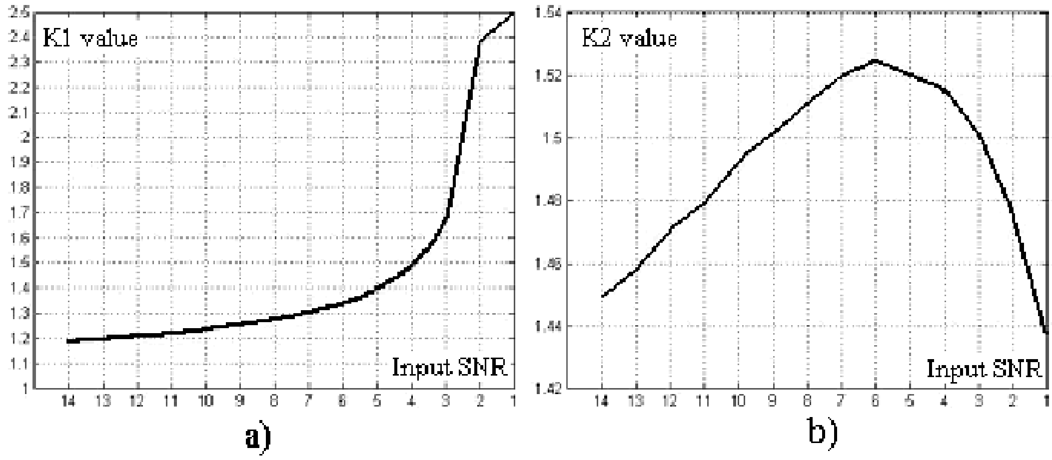

Fig. 5. The dependences of K_1 (a) and K_2 (b) values on input SNR for the noisy realization of signal (4).

denotes the correcting factor for (17) and $z_1, z_2, ..., z_N$ is the signal realization.

As it follows from expression (17), the *MMAD*-estimator accuracy should depend on K_2. It is necessary to take into consideration the following features. First, it is easy to show that for pdf (5) the median value is equal to $A/\sqrt{2}$. Then, in order to provide $\hat{A}_{MMAD} = A$ the value of the factor K_2 for noise-free signal should be equal to 1.414. However, in case of noise presence, the estimate $\hat{A}_{MMAD}$ for $K_2 = 1.414$ is commonly biased. Suppose that we determine the perfect value of K_2 as $K_2 = A/\left\langle \text{med}\left\{\left|X_{(i)} - \text{med}(z_1, z_2, ..., z_N)\right|\right\}\right\rangle$. The dependence of the perfect value of K_2 on the input SNR for the case of *Gaussian* noise is presented in Figure 5*b*. As seen, for a wide range of SNR_{inp} the perfect value of K_2 is almost constant and it is within the limits from 1.44 to 1.52.

Thus, in our simulations we apply the *MMAD*-estimate with the fixed K_2 value equal to 1.483 that corresponds to traditional *MAD* estimate [11]. One of the main reasons to do so is a lack of a priori knowledge about signal and noise characteristics. However, the proposed estimator has to perform well enough (to give almost unbiased amplitude estimate) for various SNR_{inp}. In particular, for the considered case of *Gaussian* noise, by choosing fixed $K_2 = 1.483$ we obtain the unbiased *MMAD*-estimate for $SNR_{inp} = 11$ and 2.4 and the largest absolute bias is for $SNR_{inp} = 6$ and 1.

It is worth noting that the computational complexities of these two estimators are both $O(N \log_2 N)$, since the sorting is the most demanding operation in the procedures.

IV. Accuracy analysis of the proposed signal amplitude estimators

The properties of the modified robust estimators have been examined for two test signals. As the first one, a harmonic signal was chosen: $s_1(n\Delta t) = 5\sin(2\pi C_1\ n\Delta t)$ where C_1 is a constant equal to $40/N$, $N = 512$, $\Delta t = 1$ (TS#1); the second test signal was the FM signal with linear modulation $s_2(n\Delta t) = 5\sin\left(2\pi C_2 (n\Delta t)^2\right)$ where C_2 denotes a constant equal to $0.13/N$ (TS#2).

According to the stated problem, the only available a priori information is that noise distribution is symmetric with respect to location parameter and it can possess heavy tails. To consider the performance for different noise environments, below we deal with the following three types of noise (given in accordance with increasing the tail heaviness or increasing the pdf kurtosis (K)):

- zero-mean *Gaussian* noise with variance σ_G^2 (K=0);
- noise obtained as a product of two zero-mean independent *Gaussian* random variables (*BNoise*) ($K \approx 7.47$); such kind of noise is, in particular, observed in signal waveform recon-

struction in bispectral systems [20];

- Laplacian noise (*Laplacian*) ($K \approx 7.5$);
- noise generated in the form of cube (power of three) of random independent *Gaussian* variable with zero-mean (*Cube*) [7] ($K \approx 16.4$);
- Cauchy noise (*Cauchy*) ($K \approx 30$).

One statistical parameter characterizing estimate accuracy is the bias of the amplitude estimates expressed as

$$\Delta_A = \left\langle \hat{A} \right\rangle - A. \quad (18)$$

Figure 6 depicts the dependence of $\left\langle \hat{A} \right\rangle$ on input SNR (see plots 6*a*, *b* and *d*) and noise parameters (see plots 6*c* and 6*e*) for TS#1 and TS#2 for *PEAK* and *MMAD*-estimates for five aforementioned types of noise. Averaging was performed over the ensemble of M=500 realizations.

The obtained data leads to conclusion that practically in all observed cases the amplitude estimates are characterized by a bias that can be both positive and negative. Note that Δ_A/A remains the same for given input SNR and noise pdf irrespectively of exact value of A. As seen, $\left\langle \hat{A} \right\rangle$ is almost independent on signal shape (i.e., is it a harmonic or FM signal) but it basically depends on input SNR, noise type, pdf parameters and an estimation method applied.

As expected, when *PEAK*-estimator is applied, a considerable estimate bias is observed for small input SNR and *Gaussian* noise (see Figure 6*a*). This can be explained in the following way. The contaminating noise distributions influence more significantly sample pdf shape for small input SNR values (in the considered case the sample pdf shape will tend to *Gaussian* distribution, see Figures 2*e* and *f*). As a result, with the decreasing of input SNR the position of the maximum of signal realization pdf little by little shifts towards the mean value of the affecting noise (to zero). According to this property, the *PEAK*-estimator obtains the greater negative bias with the decreasing of input SNR value and when SNR_{inp} is equal to unity the bias reaches 60% ($\left\langle \hat{A} \right\rangle \approx 0.4A$, see Figure 6*a*).

The same situation is observed for *Laplacian* and *Cauchy* noise types where with the increasing of intensity of noise (the great values of parameters λ and γ_C where λ is the parameter of *Laplacian* pdf and γ_C denotes the scale parameter of *Cauchy* distribution) the negative bias of *PEAK*-estimator reaches 20% and 40%, respectively (see plots 6*c* and *e*).

When the test signals are corrupted by *BNoise* and *Cube* noises, $\hat{A}_{PEAK}$ is characterized by positive value of bias with respect to the true value of the amplitude ($\Delta_A/A \approx 0.2$, see plots in Figures 6*b* and *c*). Such a situation results from setting fixed $K_1 = 1.2$. In other words, it is difficult to set some fixed K_1 to produce practically unbiased *PEAK*-estimates if there is no a priori information about noise type and input SNR.

The distinguishing feature of the *MMAD*-based procedure is that it produces rather stable and accurate amplitude estimation for both test signals for all investigated types of noise. The input SNR influences bias of *MMAD*-estimator only slightly. The maximum bias value is only about 6% (see the plots in Figure 6*a* for input SNR equal to unity).

Alongside with the first central moment, another important quality indicator is variance of the parameter estimates

$$\sigma^2_{est} = \frac{1}{M-1} \sum_{l=1}^{M} \left[\hat{A}_l - \frac{1}{M} \sum_{m=1}^{M} \hat{A}_m \right]^2 \quad (19)$$

where $\hat{A}_l$ is the l-th amplitude estimate.

Tables 1-3 show the amplitude estimate variances for the considered test signals depending on input SNR value and noise type. Best results are marked bold font. The obtained data analysis shows that in the case of *Gaussian* noise (see Table 1) and small input SNR (≈ 1), the *PEAK*-estimator application does not allow performing stable and accurate estimation of the signal amplitude. The ratio σ_{est}/A is of the order $[0.1, 0.25]$ and reaches 0.28 for input SNR about unity. Such a behavior can be explained by the already discussed dependence of the pdf shape on input SNR value. At the same time with the increasing the noise pdf tail heaviness, the robustness of the *PEAK*-

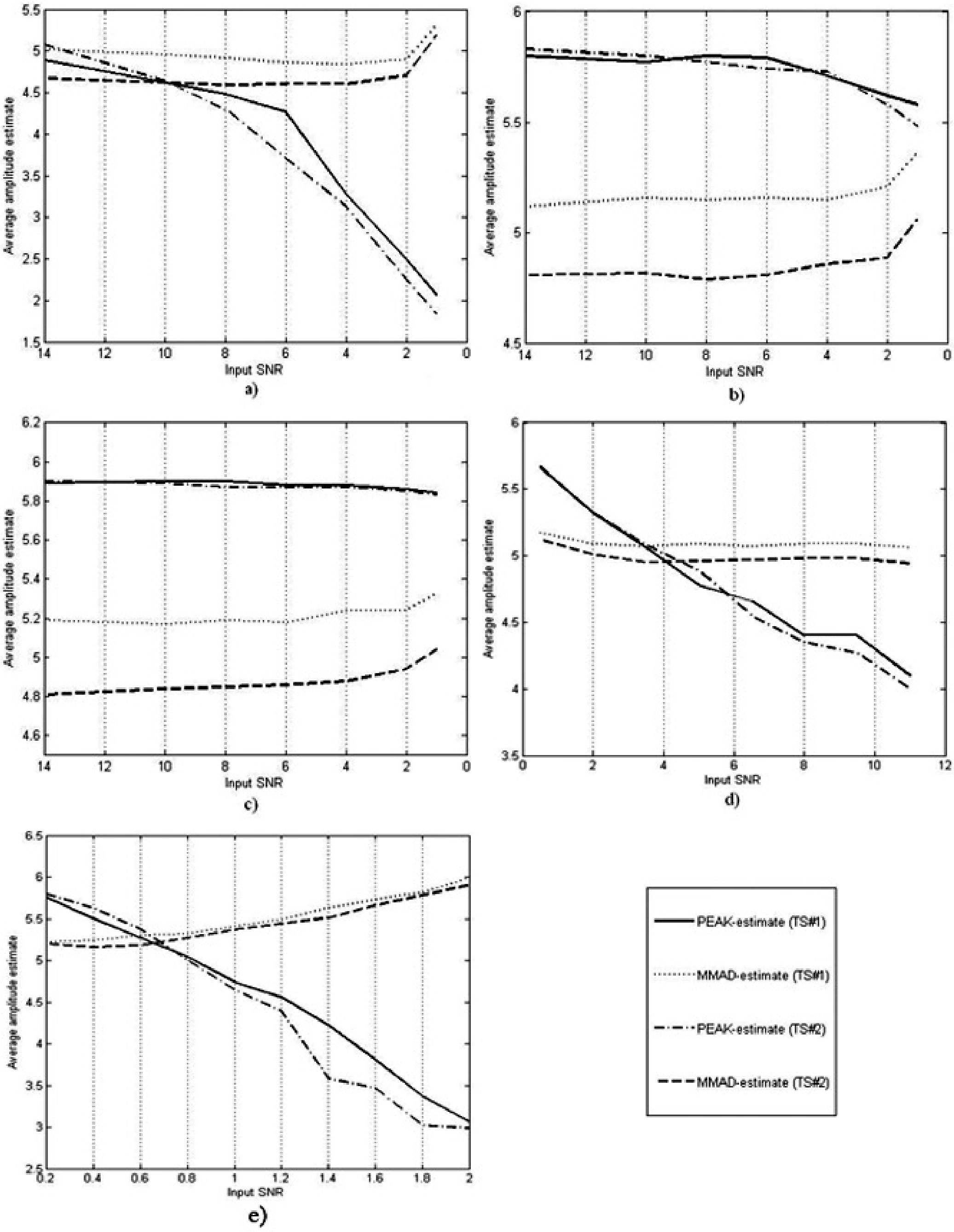

Fig. 6. Dependences of $\langle \hat{A} \rangle$ on input SNR obtained by the proposed methods in case of *Gaussian* noise (a), *BNoise* interference (b), *Laplacian* noise (c), *Cube* interference (d) and *Cauchy* noise (e).

TABLE I

THE VARIANCE DEPENDENCIES OF THE PROPOSED AMPLITUDE ESTIMATORS ON INPUT SNR VALUES

Gaussian noise								
Input SNR value		14	10	8	6	4	2	1
TS1	*PEAK*-estimator	0.260	0.600	0.600	0.980	1.54	1.610	1.88
	MMAD-estimator	**0.011**	**0.019**	**0.018**	**0.015**	**0.03**	**0.036**	**0.11**
TS2	*PEAK*-estimator	0.320	0.510	0.630	1.460	2.040	1.540	1.460
	MMAD-estimator	**0.015**	**0.012**	**0.014**	**0.020**	**0.021**	**0.035**	**0.056**
BNoise interference								
TS1	*PEAK*-estimator	0.051	0.084	0.123	0.200	0.310	0.520	2.350
	MMAD-estimator	**0.010**	**0.012**	**0.014**	**0.014**	**0.017**	**0.033**	**0.053**
TS2	*PEAK*-estimator	0.046	0.100	0.200	0.160	0.160	2.460	4.120
	MMAD-estimator	**0.008**	**0.014**	**0.015**	**0.019**	**0.022**	**0.031**	**0.038**
Cube interference								
TS1	*PEAK*-estimator	0.0051	**0.0036**	**0.0084**	**0.0011**	**0.0031**	**0.0092**	**0.0120**
	MMAD-estimator	**0.0029**	0.0059	0.0086	0.0075	0.0130	0.0110	0.0220
TS2	*PEAK*-estimator	**0.0013**	**0.0004**	**0.0006**	**0.0010**	**0.0008**	**0.0050**	**0.0016**
	MMAD-estimator	0.0130	0.0110	0.0170	0.0120	0.0220	0.0140	0.0160

TABLE II

THE VARIANCE DEPENDENCIES OF THE PROPOSED AMPLITUDE ESTIMATORS ON PARAMETER λ FOR TWO TEST SIGNALS AND LAPLACIAN NOISE

Parameter λ value		0.5	2	3.5	5	6.5	8	9.5	11
TS1	*PEAK*-estimator	0.021	0.113	0.142	0.218	0.372	0.513	0.333	0.625
	MMAD-estimator	**0.007**	**0.010**	**0.016**	**0.017**	**0.016**	**0.016**	**0.024**	**0.018**
TS2	*PEAK*-estimator	0.028	0.081	0.324	0.618	0.716	0.854	1.282	1.421
	MMAD-estimator	**0.006**	**0.015**	**0.016**	**0.017**	**0.022**	**0.020**	**0.019**	**0.026**

estimator improves. In the case of *Laplacian* noise (see Table 2) the accuracy of *MMAD*-estimator proves to be considerably better comparing to the *PEAK*-estimator. The maximal ratio σ_{est}/A for *MMAD*-estimator is equal to 0.03 for the first test signal and 0.032 for TS#2.

In the case of *Cube* noise, σ_{est}/A is smaller than 0.015 (see data in Table 1), i.e., the estimates are very accurate. When signal is corrupted by *Cauchy* (see Table 3) noise the best accuracy can be achieved by application of *MMAD*-estimator. In this case the maximal value of σ_{est}/A is approximately 0.047 and observed for great values of parameter γ_C. At the same time *PEAK*-estimator has the maximal values of $\sigma_{est}/A \approx 0.32$.

Summarizing these results, the *PEAK*-estimator accuracy considerably depends on noise pdf. For *Gaussian* noise its accuracy is poor whilst for some types of heavy-tail pdf of noise (for example, *Cube* noise) the accuracy can be excellent. Such properties in case of absence of a priori knowledge of noise pdf can be considered as a drawback of the *PEAK*-estimator.

In turn, for *MMAD*-estimator a weak dependence of σ_{est}/A on the type of noise and input SNR takes place. σ_{est}/A is commonly of the order $[0.02, 0.03]$ reaching 0.07 in the worst case of intensive *Gaussian* noise (see Table 1). Thus, the analysis of statistical characteristics of the proposed signal amplitude estimates shows that the *PEAK*-estimator is more accurate for some non-*Gaussian* (heavy-tail) noises than the *MMAD*-estimator and vice versa. At the same time, the *MMAD*-estimator has more stable operation in all considered cases. The review of properties of the considered estimators is shown in Table 4 for all considered noise environments.

Consequently, based on carried out research, one can conclude that in practice the use of the *MMAD*-estimator is preferable.

TABLE III

The variance dependencies of the proposed amplitude estimators on parameter γ_C for *Cauchy* noise

γ value		0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
	estimator										
TS1	*PEAK*	**0.009**	0.121	0.270	0.693	0.985	1.298	1.701	1.839	2.183	2.280
	MMAD	0.010	**0.013**	**0.019**	**0.021**	**0.041**	**0.033**	**0.040**	**0.043**	**0.055**	**0.057**
TS2	*PEAK*	0.032	0.099	0.587	1.119	1.929	2.075	2.791	2.756	2.578	2.714
	MMAD	**0.011**	**0.021**	**0.023**	**0.028**	**0.030**	**0.033**	**0.034**	**0.043**	**0.057**	**0.058**

TABLE IV

Comparative analysis of the proposed amplitude estimators for all considered types of noise

	Gaussian	*BNoise*	*Laplacian*	*Cube*	*Cauchy*
	PEAK-estimator				
Δ_A	increases with the decreasing of SNR$_{inp}$	excellent	differes for different SNR$_{inp}$; maximal value reaches 20%	constant; equals to 0.2A	decreases with the decreasing SNR$_{inp}$; maximal bias reaches 40%
σ^2_{est}	considerably greater than for MMAD	rapidly increases for SNR$_{inp}$<3	greater than for MMAD	equal or smaller than for MMAD	considerably greater than for MMAD
	MMAD-estimator				
Δ_A	small for all SNR$_{inp}$	tends to 0	practically unbiased	unbiased	grows with the γ_C increasing; maximal value equals to 20%
σ^2_{est}	small values	considerably smaller than for PEAK	considerably smaller than for PEAK	slightly greater than for PEAK	small values

V. Comparison of designed and known methods

FM signal filtering quality for the designed and known approaches based on scanning window nonlinear filters and optimal L-DFT (*L-DFT*) has been examined by numerical simulations. Comparative analysis has been performed for the following test signals:
1. Complex harmonic signal $s_1(t) = 5\exp(2\pi 40\, t/T)$ where $t \in [-T/2; T/2)$, T=2 is the observation interval, $\Delta t = T/N$ denotes the sampling interval and $N = 512$ is the number of samples (TS1);
2. Complex linear FM signal $s_2(t) = 5\exp\left(2\pi 60 t^2/T\right)$ (TS2).

The investigated test signals were corrupted by the following four types of noise:
- zero-mean *Gaussian* noise with variance σ^2_G;
- Laplacian noise with parameter λ_L (*Laplace*);
- Cauchy noise with parameter γ_C (*Cauchy*);
- noise with symmetrical $\alpha-$stable distribution defined by two parameters α and γ (*SaS*).

Quantitatively, the filtering performance has been evaluated by means of analysis of output SNR

$$SNR_{out} = P_S/\sigma^2_{out} \tag{20}$$

where $\sigma^2_{out} = \frac{1}{N}\sum\limits_{n=1}^{N} |\widehat{s}_F(n) - s(n)|^2$ denotes the MSE of residual fluctuations after filtering, $\hat{s}_F(n)$ is the n-th sample of output signal obtained for one of the examined methods.

One of the peculiarities of the scanning window based approach is the dependence of its properties on several factors. One of them is the applied nonlinear filter that commonly takes into account parameters of noise environment. However, in case of absence of a priori information about noise statistical characteristics, it is impossible to define the best filter type. That is why, as compromise decision that allows obtaining acceptable results both in case of *Gaussian* and non-*Gaussian* noise with different tail heaviness, the alpha-trimmed mean filter was selected. In our simulations we used the trimming parameter $N_\alpha = [N_W \cdot 0.25]$ where [...] is the integer part of a real value and N_W denotes the window size. Another important parameter affecting the performance of the *SWNF* is the size N_W. The performed simulations have shown that the smaller value of N_W the greater quality of impulse noise removal. Therefore the value of 5 have been chosen for the window size N_W. In this case the number of trimmed samples has been 2 (one maximal and one minimal values, respectively).

In robust DFT based processing approach different forms of DFT can be obtained depending on the applied estimators. One of them is the robust DFT form which is based on the robust estimators from the *L*-class. *L-DFT* method described in [13] uses the alpha-trimmed mean estimator with adoptively selected value of trimming parameter for each considered case. Thus, in situations when a priori knowledge about noise statistical characteristics partially or fully undefined such an *L-DFT* method can allow to obtain spectrum estimate close to the optimal. That is the efficiency of noise suppression can be very high.

Plots on Figures 7 and 8 present the results of filtering TS1 and TS2 corrupted by four aforementioned types of noise by means of the proposed and known methods.

Based on the obtained data we can conclude that *SWNF*-method application in the case of TS1 processing allows to perform filtering of signal in the best way almost for all considered types of noise and input SNRs in comparison to other considered methods. The exceptions are observed only in cases of:

- *Cauchy* noise (see Figure 7*c*) with great values of parameter γ_C (1.8 and 2) that can be explained by the following. The number of outliers in noise process proves to be great and as a result some of impulses remain in the signal after processing whereas, for example, *PEAK*- or *MMAD*-methods remove all of them.
- *SaS* noise described by small values of α and great γ (see Figure 7*e* and *f*). In this case the behavior of *SWNF*-method is similar to the case of Cauchy noise.
- small values of parameter λ of *Laplacian* noise model. In situation when SNR_{inp} is great, e.g., there is practically no noise, *SWNF*-method starts to introduce specific distortions such as, for example, the smoothing the signal in the polynomial extremum areas. Then σ^2_{out} increases and consequently SNR_{out} becomes smaller. In this case the use of methods based on clipping technique allow to preserve the signal shape and to perform filtering.

At the same time, the use of *SWNF* for processing the TS2 results in distortions that lead to significant decreasing of SNR_{out}. Such distortions are especially obvious in places of fast oscillations of the signal component. Then, the positive effect of noise suppression can be less than negative effect due to aforementioned distortions. As a result, SNR_{out} can be smaller than input SNR. The difference is especially large for great input SNR. Optimal *L-DFT* based filtering is able to improve the processed signal quality for all examined types of noise. However, it is worth noting that for optimal *L-DFT* the observed benefit is the smallest among all considered methods in case of TS1 (see Figure 7). At the same time, for TS2 optimal *L-DFT* outperforms the *SWNF*-method for Laplacian and *Gaussian* noise environments (see Figure 8*a* and *b*) and small values of parameter γ_C for Cauchy noise (see Figure 8*c*). Note that one of the distinctive features of the optimal *L-DFT* method is calculation complexity for large number of samples in considered interval.

Consider now the signal TS1. From Figure 7 it can be seen that the performance of the designed clipping methods is worse than for *SWNF*-method which is the best choice for TS1. Such a situation can be explained in the following way. Nonlinear filters realize filtering for all signal samples regardless of their values. In particular, they suppress noise in signal increasing/decreasing fragments [21]. At the same time, the proposed clipping methods based on *MMAD* or *PEAK*-estimators change only those samples whose magnitudes do not belong to the interval $\left[-\hat{\xi};\hat{\xi}\right]$.

At the same time, the proposed clipping methods allow to improve FM signal (TS2) for

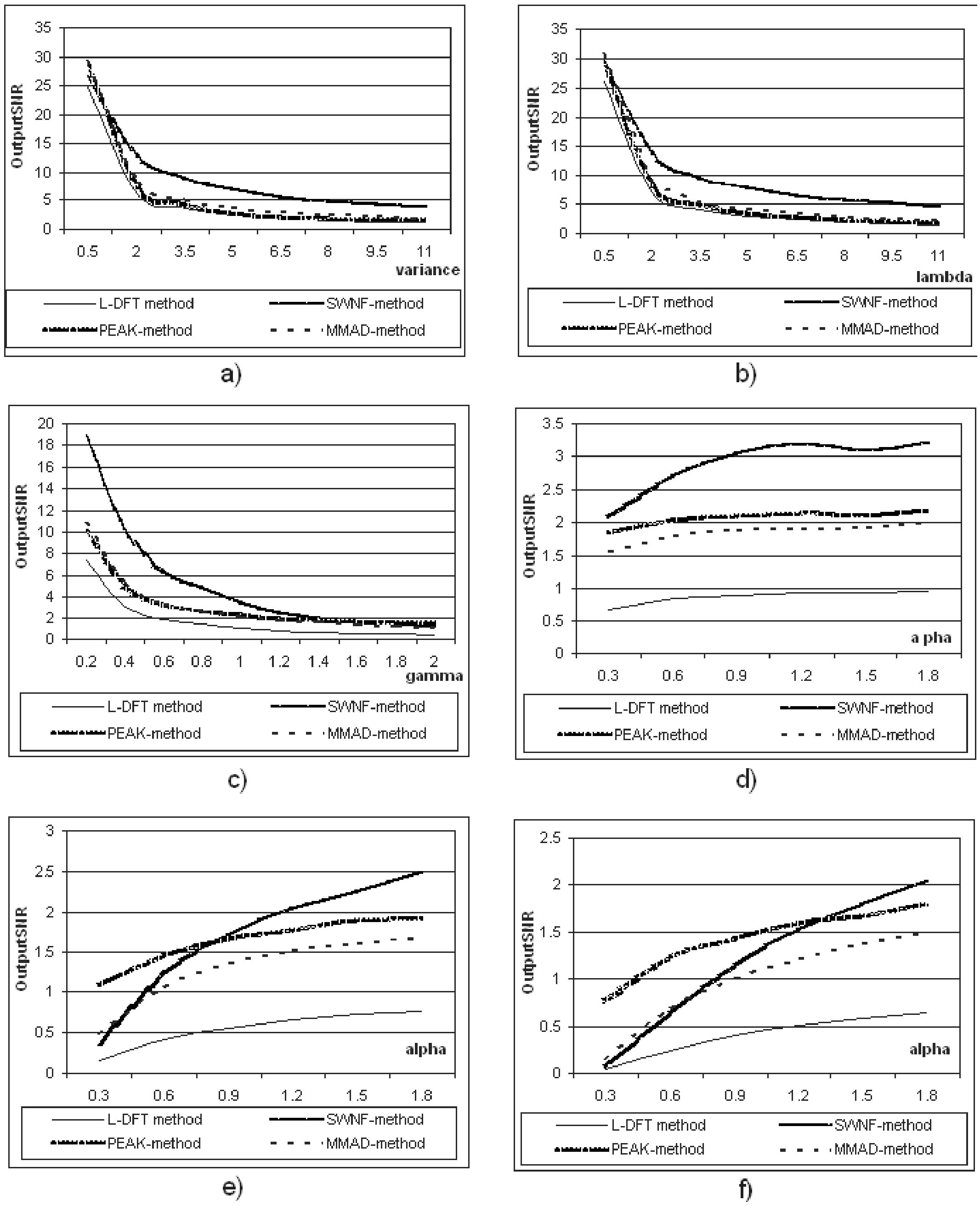

Fig. 7. Dependencies of output SNR values obtained by investigated methods for the case of TS1 on noise parameters for *Gaussian* (a), *Laplacian* (b), *Cauchy* (c) noise environments and *SaS* noise model (d, e and f) with different values of γ equal to 1.1, 1.5 and 1.9, respectively.

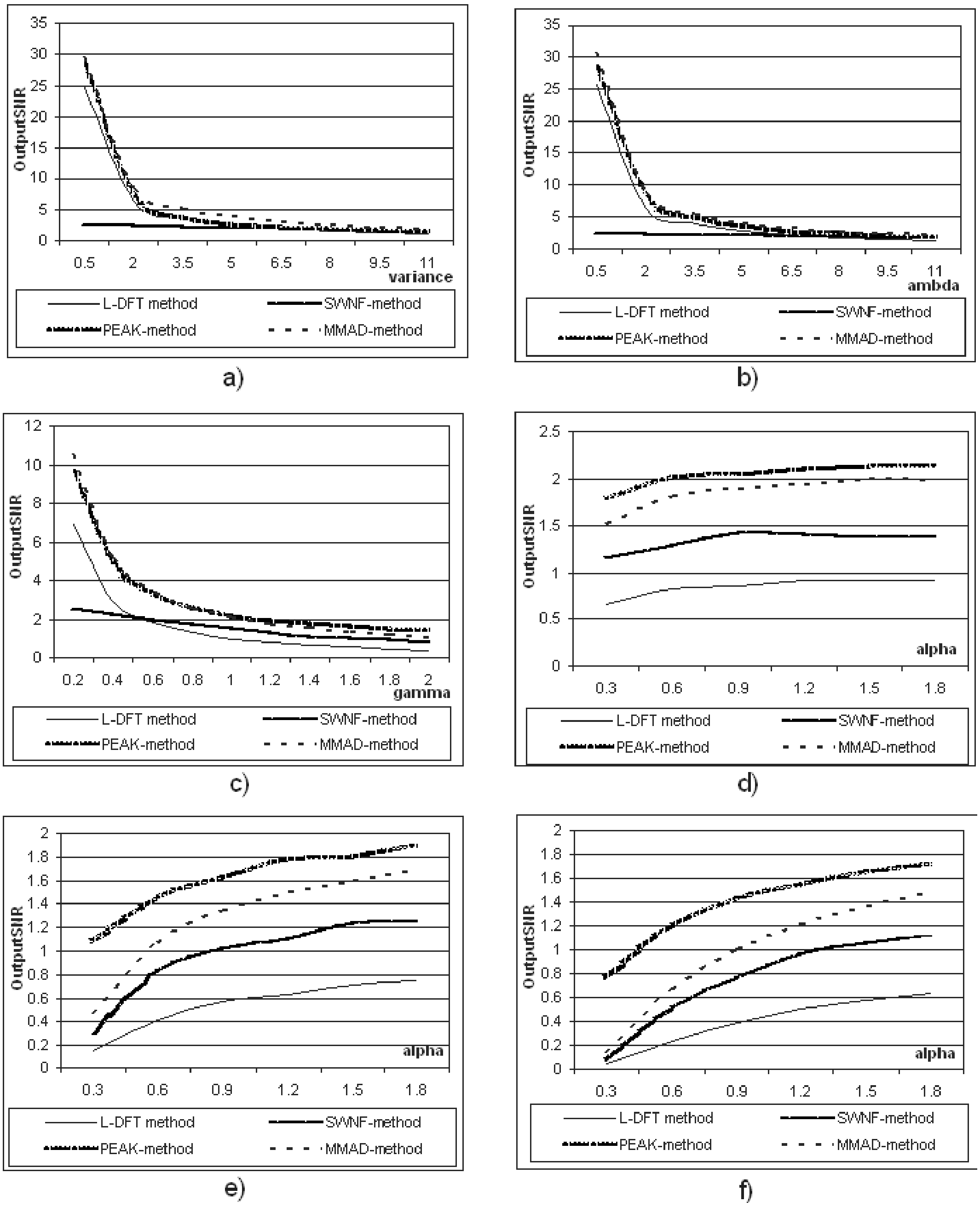

Fig. 8. Dependencies of output SNR values obtained by investigated methods for the case of TS2 on noise parameters for *Gaussian* (a), *Laplacian* (b), *Cauchy* (c) noise environments and *SaS* noise model (d, e and f) with different values of γ equal to 1.1, 1.5 and 1.9, respectively.

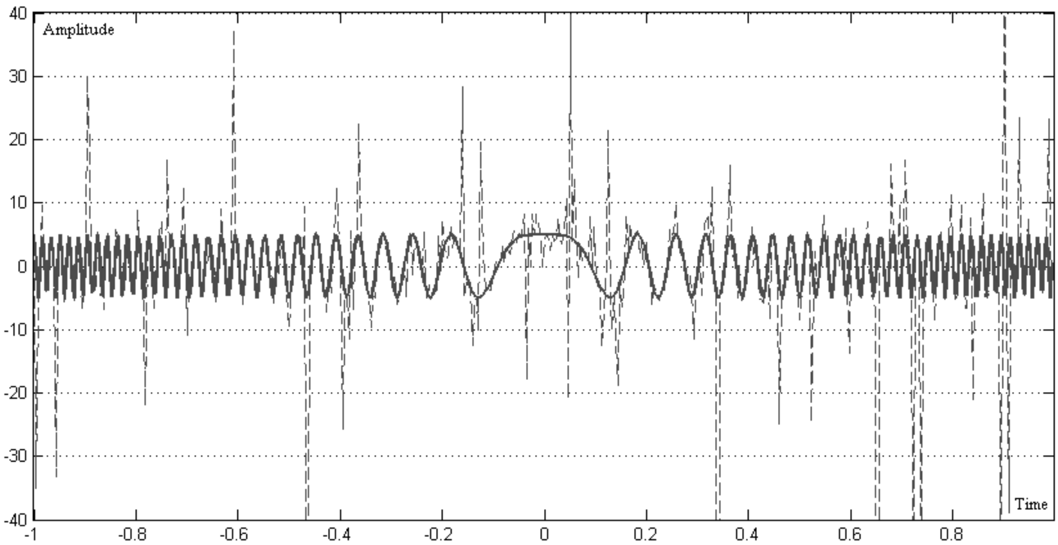

Fig. 9. Real components of the noise-free TS2 (thick solid line) and TS2 realization corrupted by *SaS* noise with parameters $\alpha = 1.5$ and $\gamma = 1.1$ and input SNR value equal to 0.02 (thin line).

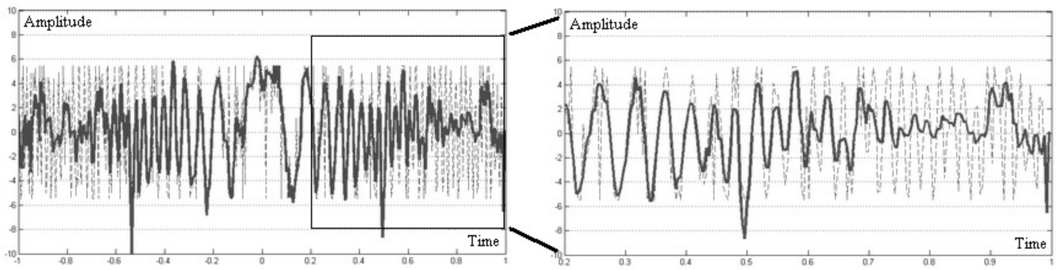

Fig. 10. Real components of TS2 estimates obtained by *SWNF*-method with $N_W = 5$ (thick solid line, SNR_{out}=1.72) and *PEAK*-method (thin line, SNR_{out}=2.24).

all examined types of noise comparing to the *SWNF* method and optimal *L-DFT* based approach. For the two designed methods, the best results (the greatest output SNR value) for *Gaussian*, *Laplacian* and *Cauchy* noise environments are obtained in case of *MMAD*-method application. Maximal benefit of the *MMAD*-method with respect to the *PEAK*-method is 25% (see Figure 8*a*, variance value is equal to 5). The worse performance of *PEAK*-method is explained by the *PEAK*-estimator properties described above in Section 3, namely, by the problem in choosing fixed K_1 and by possible large bias of *PEAK*-estimates. However the *PEAK*-method is the best choice for *SaS* noise where the maximal benefit with respect to the *MMAD*-method is about 60% and such a situation is observed for small α values and great γ (see Figure 8*e* and *f*). Let us also give a visual example of FM signal filtering by the proposed clipping technique. The real components of noise-free TS2 (thick solid line) and TS2 noisy realization corrupted by *SaS* noise with parameters $\alpha = 1.5$ and $\gamma = 1.1$ for the case of input SNR equal to 0.02 (thin solid line) are presented in Figure 9. The real components of the signal estimates obtained by clipping technique based on the proposed *PEAK*-estimator (thin solid line) and *SWNF*-method with applying alpha-trimmed mean filter (thick solid line) are demonstrated in Figure 10. For better visual analysis the parts of the signal estimates in square are represented in the greater scale. It can be seen that the *SWNF*-method considerably distorts the FM signal with suppressing the signal parts in high frequency oscillation areas. As said above, this is because the parameters of this filter should be selected accordingly to the signal spectrum and noise statistical characteristics. In the case of absence of a priori knowledge about noise characteristics the main problem of the *SWNF* approach is its inaccuracy.

At the same time, the FM signal processed by the clipping technique based on the *PEAK*-estimator is denoised well enough, SNR has been improved by about 20 dB. It is also worth noting that the results similar to the described above have been obtained for FM signals with other parameters.

VI. Conclusions

In this paper, methods for noise suppression based on the analysis of distribution of the FM signals are proposed. Two robust estimators are modified for the amplitude estimation of FM signals distorted by *Gaussian* noise or noise with heavy-tailed pdf. The first estimator (*PEAK*-estimator) is based on the determination of the minimal distance between two order statistics. Evaluation of the median of absolute deviation for data sample is the basis of the second procedure (*MMAD*-estimator). The corresponding two novel clipping methods for FM signal reconstruction are developed. The comparative analysis of the designed methods and the *SWNF* and optimal *L-DFT* based approaches is performed. It is shown that the designed methods are robust to noise influence for various noise environments. The basic advantage of the proposed methods is in the fact that they can be efficiently applied in case of limited a priori information about signal and noise properties. Besides, clipping techniques allow increasing the data processing rate comparing to optimal *L-DFT* since the former ones require much less sorting operations. The task of formulating an adaptive algorithm that estimates input SNR and noise pdf tail heaviness and then chooses the proper clipping method can be a subject of the future work.

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