# Generalization of the Fourier Domain Watermarking to the Space/Spatial-Frequency Domain

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*Abstract*— **Generalization of the Fourier domain watermarking is proposed. It has been done using joint space/spatial-frequency analysis and projections of the Wigner distribution, the two-dimensional Radon-Wigner distribution.**

## I. INTRODUCTION

Protection of information distributed through multimedia is very actual research topic in the last several years. Watermarking is one of the most important techniques for digital information protection [1]-[6]. The basic idea of watermarking is in embedding a watermark signal to image (or other multimedia signal) which will not be visible. The watermark should be detected only by using a proper private known to the copyright holder (key). A group of watermarking methods consists in embedding a watermark in transformation domain (Fourier, wavelet, DCT, Fourier-Melin). Here, it is necessary to select and change transformation coefficients so that the created changes will not have significant influence on the visual perception. The selected technique has to be robust on the geometrical distortion, filtering, histogram stretching, and other image processing operations. Having in mind the previous requests, usually, the watermark is embedded in a selected number of coefficients, avoiding low and high pass transform coefficients. The watermark signal having random amplitudes is added to the selected coefficients. Commonly, the amplitudes of watermark coefficients have zero mean Gaussian distribution. Watermark detection is performed through correlation in the selected DFT domain coefficients. Some

drawbacks of this watermarking are in its sensitivity to the high pass and nonlinear filtering.

In this paper we propose watermarking in the joint space/spatial-frequency domain. It may be considered as a generalization of the watermarking in Fourier domain or in space domain separately. A special case providing direct relation to the Fourier domain watermarking is given. Watermark detection is performed using the two-dimensional form of the Radon-Wigner distribution [7], [8].

The paper is organized as follows. Theoretical background is given in Section II. Theory is illustrated on examples in Section III, using watermarked images and various attacks. Concluding remarks are given in Section IV.

## II. WATERMARKING IN THE SPACE/SPATIAL-FREQUENCY DOMAIN

Consider an image $I(x, y)$. Fourier domain watermark can be written in the spatial domain as:

$$W_F(x, y) = \sum_{i=1}^{M} A_i \exp[j(xa_i + yb_i)] =$$

$$= \sum_{i=1}^{M} A_i S_{Fi}(x, y) \qquad (1)$$

where $A_i$ is the watermark key (randomly generated amplitudes) and $a_i, b_i$ determine location of transformation coefficients in Fourier domain. For simplicity, without loss of generality, the analytical image will be considered [9], [12]. The detection of watermark can be performed based on the value of the two-dimensional (2D) Fourier transform of $I'(x, y) = I(x, y) + W(x, y)$ at the points de-

termined by $a_i, b_i$ as follows:

$$d = \sum_{i=1}^{M} A_i FT_{2D}[I'(x,y)]_{|\text{at } (\omega_x, \omega_y)=(a_i, b_i)}$$

(2)

The previous concept of watermarking can be extended using a sum of 2D linear frequency modulated signals (chirps)

$$W_W(x,y) =$$

$$= \sum_{i=1}^{M} A_i \exp[j(a_{i1}x^2/2 + a_{i2}y^2/2 + a_{i3}xy)] \times$$

$$\exp[j(a_{i4}x + a_{i5}y)] =$$

$$= \sum_{i=1}^{M} A_i S_{Li}(x,y) S_{Fi}(x,y) \qquad (3)$$

Since the Wigner distribution is highly appropriate for processing of 2D linear frequency modulated signal, we will use it as a basic theoretical tool in this analysis. The Wigner distribution of an image $f(x,y)$ is defined as [9]-[13]:

$$WD_f(x, y, \omega_x, \omega_y) =$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x + \frac{x_0}{2}, y + \frac{y_0}{2}) \times$$

$$f^*(x - \frac{x_0}{2}, y - \frac{y_0}{2}) e^{-j\omega_x x_0 - j\omega_y y_0} dx_0 dy_0 \quad (4)$$

For signal (3) the Wigner distribution is:

$$WD_w(x, y, \omega_x, \omega_y) =$$

$$= \sum_{i=1}^{M} A_i^2 \delta(\omega_x - a_{i1}x - a_{i3}y - a_{i4},$$

$$\omega_y - a_{i2}y - a_{i3}x - a_{i5}) + cross - terms \quad (5)$$

Although the Wigner distribution of chirp signal is already completely concentrated along hyperplanes $\omega_x = a_{i1}x + a_{i3}y + a_{i4}$, $\omega_y = a_{i2}y + a_{i3}x + a_{i5}$, its application to watermark detection can be simplified and detection performance can be improved by using its projections, the Radon-Wigner distribution, along the planes defined by $a_{i1}, a_{i2}, a_{i3}$. In this way we will perform summing over delta pulses (5) along projection planes, while at the same time eliminating cross-terms [7], [8]. The projection of the Wigner distribution of chirp signal

along these projection planes will be dominant over the Wigner distribution of the image, because the second is dispersed in different projection planes. The modulus of the Radon-Wigner distribution $(RWD_i)$ can be easily calculated by multiplying the watermarked image by $A_i S_{Li}^*(x,y)$, and then calculating the Fourier transform of this product. Let us denote by $I_a(x,y)$ the watermarked and, possible, modified image. The relation for watermark detection, according to the previous consideration and (2), is given by the correlation output:

$$d = \sum_{i=1}^{M} RWD_i = \sum_{i=1}^{M} A_i \times$$

$$FT_{2D}[I_a(x,y) S_{Li}^*(x,y)]_{|\text{at } (\omega_x, \omega_y)=(a_{i4}, a_{i5})}$$

(6)

$d$ should be greater than a reference detection threshold. Watermark key can be chosen in a form of Gaussian zero mean distributed amplitudes. The variance of amplitude $A_i$ should be taken by a trade off between the watermark detection possibility and the visual imperceptibility. The location of region where the transformation coefficients $a_{ij}$, $i = 1, 2, ..., M$, $j = 1, ..., 5$ are embedded is determined according to the criteria of visual imperceptibility as well. It is possible to apply some of the techniques of visual masking on the watermarked image, such as those described in [4].

We can define a joint space/spatial-frequency watermarking that can be directly realized using the Fourier domain watermarking as a special case of the proposed watermarking method. If we take $\forall a_{i1} = a_1$, $\forall a_{i2} = a_2$, and $\forall a_{i3} = a_3$, $i = 1, 2, ..., M$, in (3) the previous watermark become:

$$W_S(x,y) = \exp[j(a_1 x^2/2 + a_2 y^2/2 + a_3 xy)] \times$$

$$\sum_{i=1}^{M} A_i \exp[j(a_{i4}x + a_{i5}y)] \qquad (7)$$

It can be realized in four steps:

1.    Generation of watermark (7) in Fourier domain using transformation coefficients $(a_{i4}, a_{i5})$, $i = 1, 2, ..., M$.

Fig. 1. First column: test images, Second column: watermarked images, Third and Fourth column: attacked watermarked images

2. Multiplication of these coefficients by 2D complex chirp $\exp[j(a_1x^2/2 + a_2y^2/2 + a_3xy)]$.

3. Watermark embedding in the image using visual masking.

4. The watermark detection procedure consists in multiplying the watermarked image by $\exp[-j(a_1x^2/2 + a_2y^2/2 + a_3xy)]$ and then detecting watermark by using $A_i$ according (6). We can embed watermark, simultaneously, in two or more hyperplanes, for example as:

$$W_S(x,y) =$$

$$= \sum_{k=1}^{K}\{\exp[j(a_{1k}x^2/2 + a_{2k}y^2/2 + a_{3k}xy)]\times$$

$$\sum_{i=1}^{M} A_{ik}\exp[j(a_{i4k}x + a_{i5k}y)]\} \qquad (8)$$

The coefficients $A_{ki}$ are determined in the same way as in the watermark defined by (7). The previous concept provides an increase in flexibility, due to parameters $(a_{1k}, a_{2k}, a_{3k})$, with respect to the Fourier domain case. For example, by taking $(a_{1k}, a_{2k}, a_{3k})$ close to zero the watermark will be more resistive to the low-pass filtering and compression. If

$(a_{1k}, a_{2k}, a_{3k})$ are significantly greater than zero then the watermark will be more resistive to high-pass filtering.

## III. EXAMPLES

The proposed watermarking is generated and tested on the standard test images Baboon (480x500), Lena (244x260) and Camera (256x256) shown in the first column of Figure 1. Watermarked images are shown in the second column of Figure 1. Watermark detection in these images is shown in Figure 2 (first column). Third and fourth columns in Figure 1 represent the attacked watermarked images. For image of Baboon we have applied: a high-pass filtering of image with cut-off frequency 1/6 of maximal frequency (third column), and a low-pass filtering of image with cut-off frequency 1/3 of maximal frequency (forth column). In the test image Lena, Gaussian white noise with variance $\sigma = 60$ is embedded (third column), as well as a radial Gaussian blur filter with $r = 4$ (fourth column). Image Camera is tested in the cases of cropping and histogram stretching.

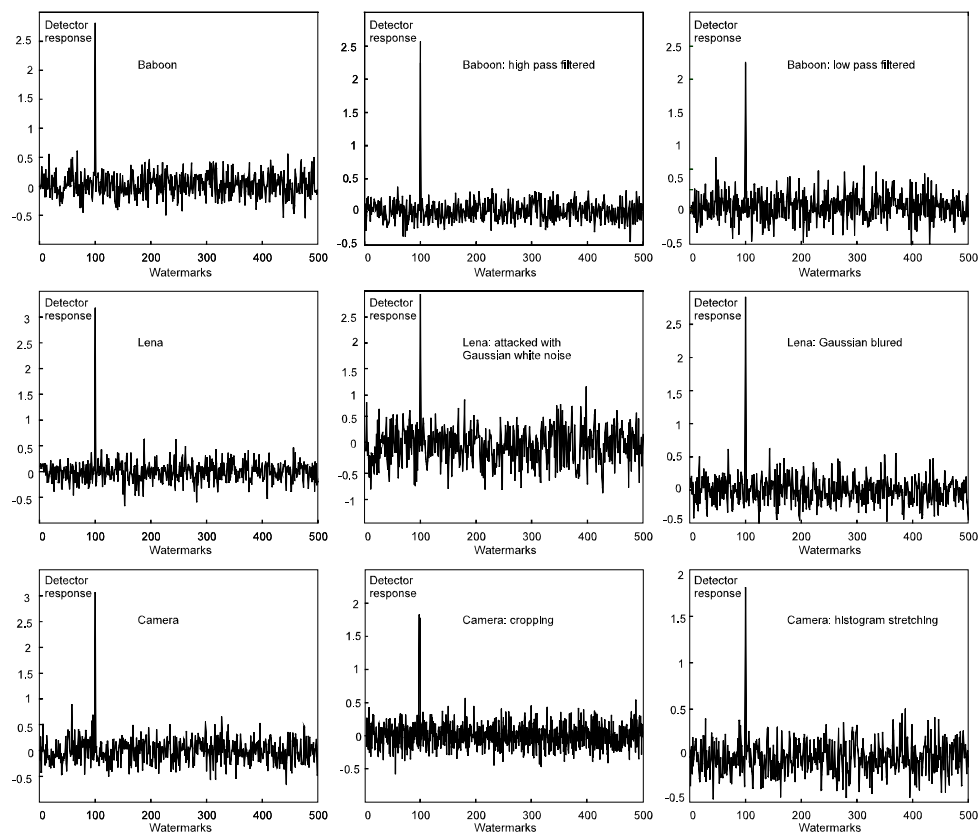Detection is performed for 500 different wa-

Fig. 2. Detection of watermark

termarks, where only the true watermark number 100 is detected. Watermark detections in the attacked images are shown in the second and third column of Figure 2.

## IV. CONCLUSION

Watermarking in the space/spatial-frequency domain is proposed, offering new possibilities in this research area. As a special case, a generalization of watermarking in the Fourier domain, is done. The presented watermarking gives more flexibility in protection algorithms than the standard watermarking in the Fourier domain. The algorithm presented here, in a combination with the method of blind watermark detection of one stronger chirp signal embedded in image, that has been presented in [11], can increase robustness to the very complex image transformations, as the rotation and scaling.

## REFERENCES

[1] G.Voyatzis, I. Pitas: "A general watermarking framework", *IEEE CGA*, Jan. 1999.

[2] Special issue on watermarking, *Signal Processing*, vol.66, no.3, 1998.

[3] I. J. Cox, J. Kilian, F. T. Leighton, T. Shamoon: "Secure spread spectrum watermarking for multimedia", *IEEE Transaction on Image Processing*, vol. 6, no. 12, Dec. 1997, pp. 1673-1687.

[4] A. Piva, M. Barni, F. Bartolini, V. Cappellini: "DCT-Based watermark recovering without resorting to the uncorrupted original image", *IEEE International Conference Image Processing '97*, Santa Barbara, CA, Oct. 1997, pp. 520-523.

[5] G. Voyatzis, N. Nikolaidis, I. Pitas: "Digital watermarking: An overview", *Proceedings of European Signal Processing Conference, EUSIPCO*, Rhode, Greece, Sept. 1998, pp. 9-12.

[6] A. Bors, I. Pitas: "Image watermarking using DCT domain constrain", *IEEE International Conference Image Processing '96*, Laussanne, Switzerland, Sept. 1996, pp. 231-234.

[7] B. Ristić, B. Boashash: "Kernel design for time-frequency signal analysis using the Radon transform", *IEEE Transaction on Signal Processing*, vol. 41, no. 5, May 1995, pp. 1996-2008.

[8] J. C. Wood, D. T. Barry: "Linear signal synthe-

sis using the Radon-Wigner distribution", *IEEE Transaction on Signal Processing*, vol. 42, no. 8, August 1994, pp. 2105-2111.

[9] Y. M. Zhu, F. Peyrin, R. Goutte: "Equivalence between the two-dimensional real and analytic signal Wigner distribution", *IEEE Transaction on Acoustic, Speech and Signal Processing*, vol. 37, no. 10, Oct. 1989, pp. 1631-1634.

[10] S. Stanković, LJ. Stanković, Z. Uskoković: "On the local frequency, group shift and cross-terms in the multidimensional time-frequency distributions; A method for multidimensional time-frequency analysis", *IEEE Transaction on Signal Processing*, vol. 43, no. 7, July 1995, pp. 1719-1725.

[11] S.Stanković, I. Djurović, I, Pitas: "Watermarking in space/spatial-frequency domain using Radon Wigner distribution", IEEE Trans. on Image Processing, submitted.

[12] J. Hormigo, G. Cristobal: "High resolution spectral analysis of images using the pseudo-Wigner distribution", *IEEE Transaction on Signal Processing*, vol. 46, no. 6, June 1998, pp. 1757-1763.

[13] S. Stanković, I. Djurović, V. Vuković: "An architecture for the realization of system for space-frequency image analysis", Electronics Letters, Nov. 1998, vol. 34, no. 23, pp. 2224-2245.