Multiwindow S-method for Instantaneous Frequency Estimation and its Application in Radar Signal Analysis

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Abstract— **A new distribution that provides high concentration in the time-frequency domain is proposed. It is based on the S-method and multiwindow approach, where different order Hermite functions are employed as multiple windows. The resulting distribution will be referred to as the multiwindow S-method. It preserves favorable properties of the standard S-method while the distribution concentration is improved by using Hermite functions of just a few first orders. The proposed distribution is appropriate for radar signal analysis, as it will be proven by experimental examples.**

I. INTRODUCTION

Various forms of time-frequency distributions have been used for the non-stationary signals analysis. Their applications cover many different fields: radars, sonars, biomedicine, image processing, etc. The simplest time-frequency representation is the short-time Fourier transform. It localizes the spectral content around a time point by using a lag window. The energetic version of this transform is spectrogram. The spectrogram is very simple for realization, but generally, it provides low time-frequency resolution. Therefore, the quadratic time-frequency distributions have been introduced in order to provide better concentration in the time-frequency domain [1]-[3]. The most frequently used among them is the Wigner distribution that provides an ideal representation for linear frequency modulated signals. However, in the case of multicomponent signals, it suffers from undesired time-frequency components called cross-terms. The S-method has been introduced to overcome this shortcoming of the Wigner distribution [4]. By using the S-method cross-terms could be completely removed while the auto-terms concentration will be the same as in the Wigner distribution. It is simple for realization and has been successfully used in radar signals analysis [5], [6], speech signals analysis [7], [8], etc.

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A new distribution based on the S-method modification is proposed in this paper. It is achieved by introducing a set of Hermite functions that improve the standard S-method concentration. The Hermite functions act as multiple windows that satisfy a number of desirable properties. The STFT is calculated by using Hermite function of a certain order, and after that, it is weighted and convolved within the frequency window. Thus, each order of Hermite function produces one form of the S-method. The cumulative distribution is obtained as their sum. In practical realizations the Hermite functions of a few first orders are sufficient to provide satisfactory results. The proposed distribution shows better properties for radar signal analysis than the standard S-method.

The paper is organized as follows. In Section II, the standard S-method is reviewed. The multiwindow S-method based on the Hermite functions is introduced in Section III. In Section IV, the efficiency of using the multiwindow S-method for obtaining high resolution radar data representation is demonstrated. Concluding remarks are given in Section V.

II. Theoretical Background – the S-method

The short-time Fourier transform (STFT) is the simplest tool for time-frequency signal representation. The STFT is obtained by sliding the window $w(t)$ along the analyzed signal $x(t)$ as follows:

$$STFT(t,\omega) = \int_{-\infty}^{\infty} x(t+\tau)w(\tau)e^{-j\omega\tau}d\tau. \quad (1)$$

The spectrogram is defined as a square module of the STFT:

$$SPEC(t,\omega) = |STFT(t,\omega)|^2. \quad (2)$$

However, in most cases the spectrogram cannot provide high time-frequency resolution. The significant improvement of the time-frequency resolution is achieved by using the quadratic time-frequency distributions, such as the Wigner distribution.

In order to produce the same auto-terms concentration as in the Wigner distribution but to remove the cross-terms, the S-method has been introduced. The appropriate time-frequency representation is obtained by combining the values of the STFT along the frequency axis as follows [4]:

$$SM(t,\omega) = \\ = \int_{-\infty}^{\infty} P(\theta)STFT(t,\omega+\theta)STFT^*(t,\omega-\theta)d\theta, \quad (3)$$

where $P(\theta)$ represents a finite frequency domain window function. The discrete version of the S-method is given by:

$$SM(n,k) = \\ = \sum_{l=-L}^{L} P(l)STFT(n,k+l)STFT^*(n,k-l), \quad (4)$$

where n and k are the discrete time and frequency variables, respectively, while $P(l)$ is the window of the length $2L+1$. By taking the rectangular window, the discrete S-method can be written as:

$$SM(n,k) = |STFT(n,k)|^2 + \\ +2Re\left\{\sum_{l=1}^{L} STFT(n,k+l)STFT^*(n,k-l)\right\}. \quad (5)$$

Note that the terms in summation improve the quality of spectrogram toward the quality of the Wigner distribution. The window $P(l)$ should be wide enough to enable the complete summation over the auto-terms. At the same time, in order to remove the cross-terms, it should be narrower than the minimal distance between the auto-terms. The convergence within $P(l)$ is very fast, providing a high auto-terms concentration with only a few summation terms. In many applications $L<10$ can be used [4], [6]-[8].

III. Multiwindow S-method based on the usage of Hermite Functions

An efficient solution for the time-varying spectrum estimation is achieved by introducing multiwindow analysis methods [9]-[12]. In order to minimize the variance, these methods are mainly based on the usage of orthogonal windows. At the same time, the windows should be optimally concentrated to provide low bias estimate. Thus, for an efficient time-frequency analysis, the multiple orthogonal windows that are optimally concentrated in the joint time-frequency domain should be employed. Hence, the idea is to use a set of orthogonal Hermite functions that are computationally localized in both time and frequency domains. Particularly, they are optimally localized in the circular time-frequency region: $\{(t,\omega):\ t^2+\omega^2 \le R^2\}$, of area πR^2 [12].

The k-th order Hermite function is defined as:

$$\Psi_k(t) = \frac{(-1)^k e^{t^2/2}}{\sqrt{2^k k!\sqrt{\pi}}} \cdot \frac{d^k(e^{-t^2})}{dt^k}. \quad (6)$$

Although the computation of Hermite functions seems to be a demanding task, these functions could be easily obtained by using recursive realization as follows [13], [14]:

$$\Psi_0(t) = \frac{1}{\sqrt[4]{\pi}}e^{-t^2/2},$$

$$\Psi_1(t) = \frac{\sqrt{2}t}{\sqrt[4]{\pi}}e^{-t^2/2},$$

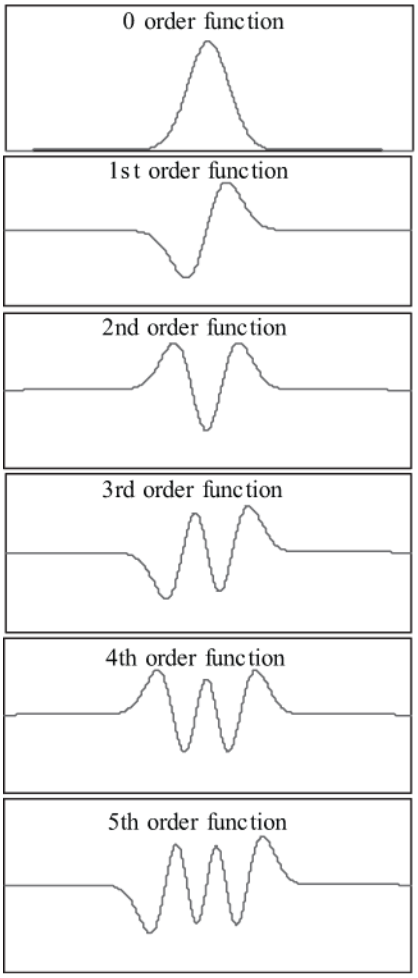

Fig. 1. Illustration of Hermite functions $\Psi_k$for k=0,1,2,3,4,5.

$$\Psi_k(t)=t\sqrt{\frac{2}{k}}\Psi_{k-1}(t)-\sqrt{\frac{k-1}{k}}\Psi_{k-2}(t),\ \forall k\geq 2. \tag{7}$$

Note that the zero order Hermite function Ψ_0 corresponds to Gaussian window function. Hermite functions vanish at the infinity and represent the eigenfunctions of the Fourier transform. Due to their good properties, these functions have been used in various applications: texture analysis, projection filtering, image foveation, speech processing, etc. [13]-[16]. Hermite functions for the first six orders (Ψ_k, k=0,1,2,3,4,5) are shown in Fig 1.

Starting from the idea introduced by Thomson [9], the multiple windows spectrogram has been introduced in [10]. It is defined as a weighted sum of K spectrograms, where K is the total number of windows, i.e. Hermite functions used in the realization. The multiwindow spectrogram is defined as:

$$MWS_K=\sum_{k=0}^{K-1}d_k(t)SPEC_k(t,\omega)=$$
$$=\frac{1}{2\pi}\sum_{k=0}^{K-1}d_k(t)\left|\int x(\tau)\Psi_k(\tau-t)e^{-j\omega\tau}d\tau\right|^2. \tag{8}$$

where $x(t)$ is signal, while $d_k(t)$ are the weighting coefficients. By using K Hermite functions, it is possible to remove K-1 phase derivatives: 2^{nd}, 3^{rd},..., and K-th. Thus, higher number K of the employed functions provides higher concentration in the time-frequency plane. It has been shown that the multiwindow spectrogram outperforms the standard one for various types of signals, and it represents a very efficient tool for instantaneous frequency estimation [11]. Similarly, as in the case of spectrogram, the multiwindow concept can be extended to the S-method, as well.

By analogy with the standard S-method, the multiwindow S-method can be defined as a convolution of Hermite function based STFT. Namely, after calculating the STFTs by using Hermite functions of different orders, we perform the convolution of the STFTs for each order k within an additional frequency window $P(\theta)$. Finally, the multiwindow S-method is obtained as a weighted sum of the convolution terms, as follows:

$$MWSM_K(t,\omega)=$$
$$=\sum_{k=0}^{K-1}\int_\theta P(\theta)d_k(t)STFT_k(t,\omega+\theta)\times$$
$$\times STFT_k^*(t,\omega-\theta)d\theta, \tag{9}$$

where $STFT_k(t,\omega)$ denotes the short-time Fourier transform obtained by using the k-th order Hermite function:

$$STFT_k(t,\omega)=\int x(\tau)\Psi_k(t-\tau)e^{-j\omega\tau}d\tau. \tag{10}$$

The frequency window $P(\theta)$ has the same properties as in the case of the standard S-method. The discretization of (9) leads to the form that is suitable for practical applications and can be written as:

$$MWSM(n,k) = \sum_{k=0}^{K-1} d_k(n) \left| MWS_k(n,k) \right|^2 + \\ + \sum_{k=0}^{K-1} 2Re \left\{ \sum_{l=1}^{L} d_k(n) P(l) STFT_k(n,k+l) \times \right. \\ \left. \times STFT_k^*(n,k-l) \right\}. \quad (11)$$

It can be observed that the first term within the multiwindow S-method represents the multiwindow spectrogram. The convolution terms improve the quality and resolution of multiwindow spectrogram. For the N-th order polynomial phase signal ($N>K$), the multiwindow S-method contains (N-K)/2 phase derivatives (higher than the K-th derivative), which is half of their number in the case of multiwindow spectrogram. The spread factors for the spectrogram, the S-method, the multiwindow spectrogram and the multiwindow S-method are given in Table I. It is also important to note that the multiwindow S-method provides better resolution comparing to the standard S-method, similarly as the multiwindow spectrogram outperforms the standard one.

An example demonstrating the efficiency of the multiwindow S-method is given in Fig 2. The signal is considered in the form: $x(t) = \exp(6\pi j \cos(0.1\pi t) + 4\pi j \cos(0.2\pi t))$.

The results obtained by using the spectrogram, the S-method, the multiwindow spectrogram and the multiwindow S-method are presented (Figs 2.a, b, c and d, respectively). The multiwindow distributions are obtained by using four Hermite functions (k=0,1,2,3). The parameter L=5 is used in the calculation of standard and multiwindow S-method. Observe that the multiwindow S-method in Fig 2.d provides better concentration than the other considered distributions. Also, the computational demands for the multiwindow S-method are just slightly increased compared to the standard S-method having in mind that satisfactory results are achieved with a small number of Hermite functions.

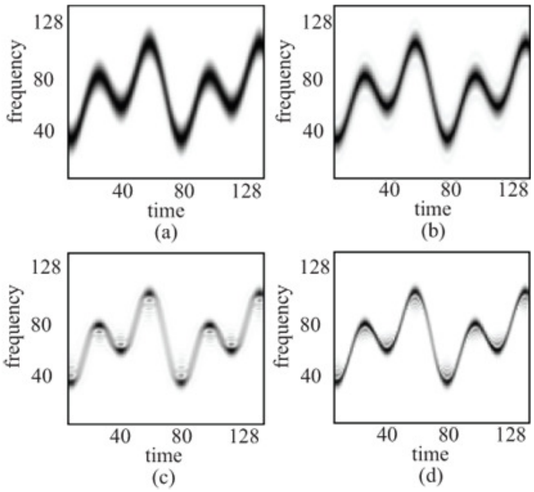

Fig. 2. Time-frequency distributions of a signal $x(t)$: a) Spectrogram, b) S-method by using L=5, c) multiwindow spectrogram by using four Hermite functions, d) multiwindow S-method by using four Hermite functions and L=5

Based on the considered distributions, the instantaneous frequency is estimated in the presence of noise for several values of signal to noise ratio: SNR=5dB, SNR=7.5dB, and SNR=10dB. Mean square errors (MSE) are calculated in 40 realizations, while the average MSE is calculated as:

$$E\{MSE\} = \frac{1}{N} \sum_{n=0}^{N-1} E\left\{ \left[f(n) - \overline{f}(n) \right]^2 \right\},$$

where $f(n)$ represents the true instantaneous frequency while $\overline{f}(n)$ is the instantaneous frequency estimated by using the considered time-frequency distributions (TFD): $\overline{f}(n) = \max_m TFD(n,m)$.
The results are presented in Table II. An illustration of the mentioned time-frequency distributions for a noisy signal: $y(t) = x(t) + \nu(t)$, is given in Fig 3 (for SNR=7.5dB).

Observe that the multiwindow Hermite S-method is less sensitive to noise than the other presented distributions. This is an additional advantage, especially in the case of real signals, that are usually corrupted by the noise.

Additionally, it is important to emphasize that the Hermite S-method provides cross-terms free representation for multicomponent

TABLE I
Spread factors for some time-frequency distribution

Distribution	Spread factor	
Spectrogram	$Q(t,\tau)=\phi^2(t)\frac{\tau^2}{2!}+\phi^3(t)\frac{\tau^3}{3!}+\phi^4(t)\frac{\tau^4}{4!}+...$	
S-method	$Q(t,\tau)=\phi^3(t)\frac{\tau^3}{2^2 3!}+\phi^5(t)\frac{\tau^5}{2^4 5!}+...$	
Multiwindow Spectrogram with K functions	$Q(t,\tau)=\phi^{(K+1)}(t)\frac{\tau^{K+1}}{(K+1)!}+\phi^{(K+2)}(t)\frac{\tau^{K+2}}{(K+2)!}+\phi^{(K+3)}(t)\frac{\tau^{K+3}}{(K+3)!}+...$	
Multiwindow S-method with K functions	even K	$Q(t,\tau)=\phi^{(K+1)}(t)\frac{\tau^{K+1}}{2^K(K+1)!}+\phi^{(K+3)}(t)\frac{\tau^{K+3}}{2^{(K+2)}(K+3)!}+...$
	odd K	$Q(t,\tau)=\phi^{(K+2)}(t)\frac{\tau^{K+2}}{2^{K+1}(K+2)!}+\phi^{(K+4)}(t)\frac{\tau^{K+4}}{2^{(K+3)}(K+4)!}+...$

TABLE II
Mean square error of the instantaneous frequency estimation for different values of snr

Mean square Error (MSE)	SNR =5dB	SNR =7.5dB	SNR =10dB
Spectrogram	45.41	40.05	32.77
S-method	40.33	31.34	21.9
Multiwindow Spectrogram	24.17	20.56	13
Multiwindow S-method	13.77	12.9	11.8

signals in the same manner as the standard S-method. Again in this case, the width $2L+1$ of frequency domain window $P(\theta)$ should be narrower than the minimal distance between the auto-terms. An illustration of Hermite S-method for multicomponent signal: $z(t)=\exp(3\pi j\cos(0.1\pi t)+2\pi j\cos(0.2\pi t)+12jt)+\exp(4\pi j\cos(0.15\pi t)+4\pi j\cos(0.05\pi t)-12jt)$, is given in Fig 4.

The optimal weighting coefficients $d_k(n)$ in (11) are calculated according to [11]:

$$\sum_{k=0}^{K-1} d_k(n)\frac{\sum_{m=-N/2}^{N/2-1}A^2(n+m)\Psi_k^2(m)m^i}{\sum_{m=-N/2}^{N/2-1}A^2(n+m)\Psi_k^2(m)}=$$

$$=\left\{\begin{array}{ll}1, & i=0\\ 0, & i>0\end{array}\right., \quad i=0,1,\ldots,K-1, \quad (12)$$

where $A(n)$ is the signal amplitude, while N is the number of samples within the window function. Note that for constant amplitude within the window ($A(n+m)=A(n)$), the weighting coefficients are constants, given in Table III.

IV. Application of Multiwindow S-method for highly concentrated representation of radar signals

Efficiency of the multiwindow S-method will be also proven in the example with radar data. Hence, we provide a brief overview of the basic radar signal model and related target motion estimation.

Consider a Doppler radar transmitting a signal in the form [17], [18]:

$$s(t)=e^{j\pi\frac{\omega_s}{T_r}t^2}, \quad 0\le t\le T_r, \qquad (13)$$

where T_r is the repetition time. Commonly, the transmitted signal consists of M chirps:

$$s_M(t)=e^{-j\omega_0 t}\sum_{m=0}^{M-1}s(t-mT_r), \qquad (14)$$

where ω_0 is the radar operating frequency. Note that we can consider just one component of the received signal $s_m(t)=e^{-j\omega_0 t}s(t-mT_r)$. Furthermore, if the target distance from radar (also known as range) is $d(t)$, the received signal is delayed with respect to the transmitted signal for $2d(t)/c$:

TABLE III
WEIGHTING COEFFICIENTS d_k FOR CONSTANT AMPLITUDE SIGNAL FOR K=1,...,11

	$\mathbf{d_0}$	$\mathbf{d_1}$	$\mathbf{d_2}$	$\mathbf{d_3}$	$\mathbf{d_4}$	$\mathbf{D_5}$	$\mathbf{d_6}$	$\mathbf{d_7}$	$\mathbf{d_8}$	$\mathbf{d_9}$	$\mathbf{d_{10}}$
K=1	1										
K=2	1.5	-0.5									
K=3	1.75	-1	0.25								
K=4	1.875	-1.375	0.625	-0.125							
K=5	1.937	-1.625	1	-0.375	0.062						
K=6	1.968	-1.781	1.312	-0.687	0.219	-0.031					
K=7	1.984	-1.875	1.546	-1	0.453	-0.125	0.016				
K=8	1.992	-1.929	1.710	-1.273	0.727	-0.289	0.070	-0.008			
K=9	1.996	-1.961	1.820	-1.492	1	-0.507	0.179	-0.039	0.003		
K=10	1.998	-1.978	1.890	-1.656	1.246	-0.754	0.344	-0.109	0.021	-0.002	
K=11	1.900	-1.561	0.955	-0.223	-0.357	0.573	-0.460	0.237	-0.079	0.016	-0.001

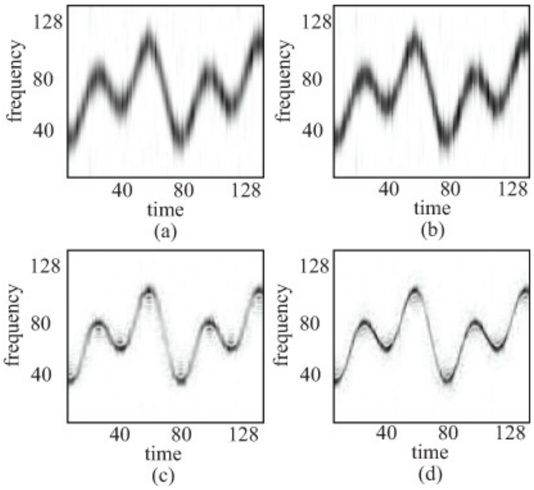

Fig. 3. Time-frequency distributions of a noisy signal y(t): a) Spectrogram, b) standard S-method, c) multiwindow spectrogram, d) multiwindow S-method

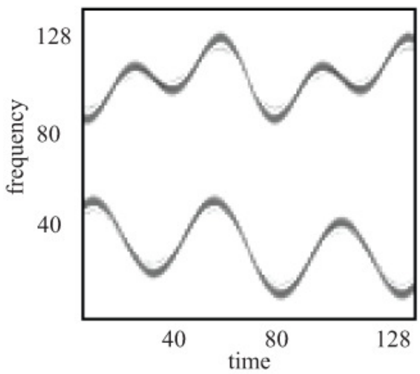

Fig. 4. An illustration of the Hermite S-method in the case of multicomponent signal

$$s_m(t) = \sigma e^{-j\omega_0\left(t-\frac{2d(t)}{c}\right)} s(t - \frac{2d(t)}{c} - mT_r), \tag{15}$$

where c is the propagation rate equal to the speed of light, while σ is the reflection coefficient.
The Doppler phase can be written as [17]:

$$\phi_r(t) = \int_0^t \Delta\omega_d(\tau)d\tau, \tag{16}$$

where $\Delta\omega_d$ is the Doppler frequency shift.
Thus, the phase of the received signal changes as follows:

$$\phi_r(t) = \frac{4\pi}{\lambda} d(t) = \frac{2\omega_0}{c} d(t), \tag{17}$$

where λ is the transmitted signal wavelength.

According to (15) and (17), the Doppler frequency shift will be proportional to the target radial velocity:

$$\Delta\omega_r(t) = \frac{d}{dt}\left[\phi_r(t)\right] = \frac{2\omega_0}{c}\frac{d}{dt}\left[d(t)\right] = \frac{2\omega_0}{c} v(t). \tag{18}$$

The radar target may contain structures that produce mechanical vibrations or rotations, causing frequency modulation in the returned signal. This modulation is known as micro-Doppler phenomenon. The vibration of a reflecting surface may be measured with the phase change. Consequently, the Doppler frequency shift can be used to detect vibrations of structures on a target [18], [19].

The target can be observed as a set of primary reflecting points i.e. point scatterers [19].

Assuming that the vibrating scatterer is set to a radian frequency oscillation of ω_ν, its time-varying phase changes according to [19]:

$$\phi(t) = \frac{4\pi D_\nu}{\lambda} \sin(\omega_\nu t), \qquad (19)$$

where D_ν is amplitude of the vibration and λ is the wavelength of the transmitted signal. The micro-Doppler frequency induced by the vibration can be obtained as:

$$f_D(t) = \frac{1}{2\pi} \frac{d\phi}{dt} = \frac{2D_\nu \omega_\nu}{\lambda} \cos(\omega_\nu t). \qquad (20)$$

The micro-Doppler signature caused by the vibrating structure is important for target identification, such as for example, human walking gait. The time-frequency signatures could be very suitable for this purpose, since the micro-Doppler of the vibrating scatterer is a time-varying frequency spectrum [18], [19].

Example: In this example we have considered a radar signal that captures human movements. The signal contains multiple physical movements taking place simultaneously. Various body parts have different shifts, since they are moving with various velocities. The strongest component corresponds to the main body movements, while the other components correspond to swinging arms and legs, etc. For example, the swinging arms induce frequency modulation of the returned signal and generate side-bands about the body Doppler.

The standard spectrogram and the S-method are calculated by using Hanning window and they are presented in Fig 5.a and Fig 5.b, respectively. Also, the spectrogram and the S-method obtained by using only one Hermite function (zero order function that corresponds to Gaussian window) are considered in Fig 5.c and Fig 5.d, respectively. The time domain windows of length 256 are used.
The multiwindow spectrogram is shown in Fig 5.e, while the multiwindow S-method is presented in Fig 5.f. The multiwindow time-frequency representations are calculated by using five Hermite functions. Both the standard and the multiwindow S-method are calculated by using L=3, which provides cross-terms free representation with good concentration of auto-terms. It has been shown that this is the optimal value for considered application. Higher value of parameter L would produces cross-terms, whose value would increase as L increases. Namely, after L has reached the value equal to the distance between the auto-terms, cross-terms start to appear.

It is interesting to note that the standard S-method improves the concentration in comparison with the spectrogram. Some further improvements are achieved by using the multiwindow spectrogram. However, the multiwindow S-method outperforms both the standard S-method and the multiwindow spectrogram, which is especially emphasized in the region that corresponds to torso component. The standard S-method and multiwindow spectrogram can provide sufficiently good concentration for linear or almost linear frequency components. However, the Hermite S-method can further improve the concentration of components with higher nonlinearity.

Note that good concentration of the Hermite S-method is achieved by using just a few orders (k=0,1,2,3,4) of Hermite functions. Namely, the presence of the fifth and higher order phase derivatives is not significant. Thus, higher number of Hermite functions (that would remove them) does not contribute to the concentration improvement.

V. Conclusion

The multiwindow S-method is developed from the idea to merge two efficient signal processing tools, the S-method and the Hermite functions, into a modified, highly concentrated time-frequency distribution. It exhibits good properties in the presence of noise, while regarding the concentration, outperforms both the standard S-method and the multiwindow spectrogram. This method provides very satisfying results even by using small number of Hermite functions and small number of samples within the frequency domain window. Therefore, it is computationally effective and suitable for realization. The efficiency of the proposed method is proven on the analytic signal, as well as on radar data.

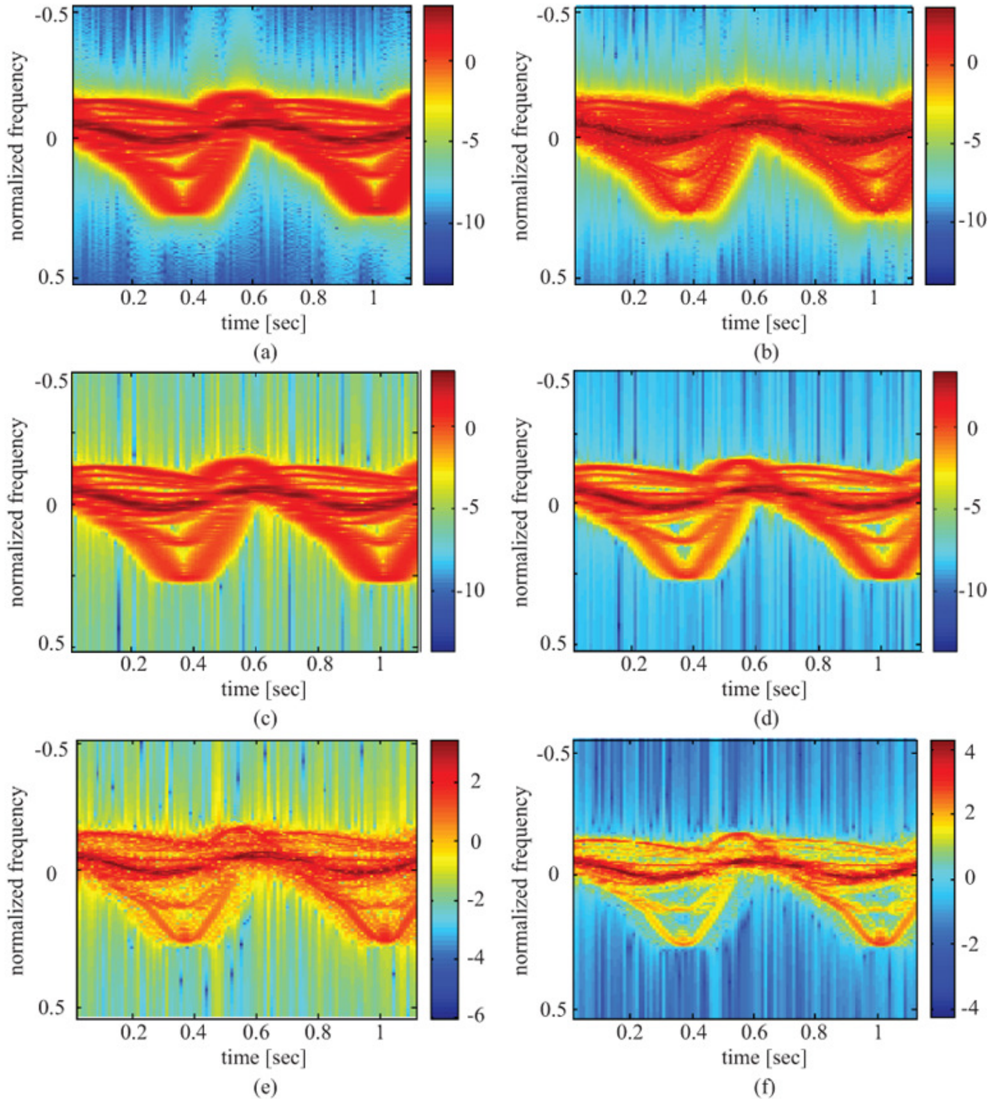

Fig. 5. Time-frequency representations of radar signal: a) Standard spectrogram with hanning window, b) standard S-method by using hanning window, while L=3; c) Spectrogram with one Hermite function, d) S-method with one Hermite function, e) Multiwindow spectrogram with 5 Hermite functions, d) Multiwindow S-method with five Hermite functions and L=3

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