

# An Implementation of the L-estimate Distributions for Analysis of Signals in Heavy-Tailed Noise

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*Abstract*— An analog-digital system for implementation of the robust time-frequency distributions is proposed. It provides an efficient real-time analysis of nonstationary signals corrupted by heavy-tailed noise. The most important part of this system is the realization of the L-estimate short-time Fourier transform that requires sorting operation. The proposed solution is designed to execute sorting operation within a single clock cycle, regardless of the number of inputs. The performance of the proposed hardware is tested on a real signal example.

## I. INTRODUCTION

Time-frequency signal analysis has been intensively developed in the last two decades. Consequently, the time-frequency based algorithms have been used in various practical applications that assume non-stationary signals with time-varying spectrum. These applications are usually related to audio and speech signal analysis [1]; radar, sonar, digital image, and digital video processing; and compression [2], [3]. Furthermore, the time-frequency analysis has been applied successfully in on-line monitoring and detection of welding defects [4], seismic surveying, system identification, medical instrumentation, etc.

Depending on the nature of analysed signals, various time-frequency distributions have been proposed [5]-[8]. They need to satisfy different application-specific requirements related to the energy concentration, robustness to noise, marginal properties. By increasing the number of those requirements, the design of time-frequency distributions become a more complex and more computationally demanding task. Hence, their software realization can-

not be used in many fast real-time applications. In order to increase their applicability, and to provide a real-time implementation of time-frequency distributions, various hardware solutions have been proposed [1]-[4], [9]-[14]. They have been designed for standard distributions, such as the standard spectrogram [1]-[3] that has been used, for example, in speech signal processing. In addition, an efficient FPGA implementation of the standard Wigner distribution combined with different kernels, has been proposed in [4]. Note that the standard distributions are optimal only in the presence of Gaussian noise. However, the signals are often corrupted by a heavy-tailed noise (speckle noise, salt and pepper noise, clutters, etc). In this case, the standard distributions produce poor results and the robust distributions should be used instead. They can be obtained either by using the recursive realization or by using the concept of L-estimates. The recursive realization of the robust distributions is based on their standard counterparts [15], [16]. Thus, the existing hardware solutions [2]-[4], [11]-[13] could be used sequentially to perform each of the iterations in the recursive realization. Note that the distribution is recalculated at each iteration. However, due to the large number of iterations required in order to provide satisfactory results, this approach is not appropriate for fast real-time applications. On the other hand, the L-estimate form of robust time-frequency distributions [17] is computationally far less demanding and faster than the recursive realization. Furthermore, the L-estimate approach can produce different distribution forms depending on the noise statistics. It means that the hardware for the L-estimate approach can

be used for both the robust and the standard distributions, i.e., it can be used in the presence of different impulse noises, but also in the presence of Gaussian noise. Having in mind this universality of application, in this paper we consider the hardware implementation of the L-estimate time-frequency representation.

Fast analog-digital hardware for robust time-frequency signal analysis is first proposed. The analog circuits are used to speed up the computationally demanding operations, thus providing faster execution time when compared to digital hardware (implemented in FPGA). The main part of the proposed system is a circuit for realization of the L-estimate short-time Fourier transform (STFT). Here, the most complex and computationally most demanding is the sorting operation. Various realizations of sorting operation have been proposed [18]-[21]. However, most of them are complex and their execution time becomes significant by increasing the number of inputs. For efficient realization of sorting operation, an analog-digital sorter with execution time of  $O(1)$  order is proposed. The comparison results are fed to the digital combination network to reorder the input sequence into the sorted one. Next, the L-estimate STFT is obtained as a sum of the sorted elements multiplied by the corresponding weights. The proposed solution is tested on a real chirp signal corrupted by mixed Gaussian and impulse noise. The performance is compared with the software realization and the digital FPGA realization based on the fast Bitonic sort algorithm.

The paper is organized as follows. The basic theory of the L-estimate form of time-frequency representations is presented in Section II. The realization of the proposed system is given in Section III. Section IV is devoted to the performance analysis of the proposed hardware implementation. Concluding remarks are given in Section V.

## II. THEORETICAL BACKGROUND

The simplest and the most commonly used time-frequency representation is the STFT. In general, the STFT can be obtained as a solution of the optimization problem:

$$STFT(n, k) = \arg \min_{\mu} \sum_{m=-N/2}^{N/2-1} F(e(n, k, m)), \quad (1)$$

where  $F(e)$  is the loss function, while  $e(n, k, m)$  is the error function:

$$e(n, k, m) = x(n + m)e^{-j2\pi mk/N} - \mu. \quad (2)$$

The standard STFT follows as a solution of (1) for the squared absolute loss function  $F(e)=|e|^2$ , which is obtained from the maximum likelihood estimation theory, for the case of Gaussian noise. The absolute loss function  $F(e)=|e|$  is a maximum likelihood estimate in the presence of Laplacian noise, and it produces the robust STFT. Furthermore, according to Huber estimation theory, the loss function  $F(e) = |Re(e)| + |Im(e)|$  should be used in the case of mixed Gaussian and impulse noise. By combining this loss function with the concept of L-estimates, the L-estimate robust STFT can be defined as [17]:

$$STFT_L(n, k) = \sum_{i=1}^N a_i(rs_i(n, k) + j \cdot is_i(n, k)), \quad (3)$$

$$rs_i(n, k) \in Rs(n, k), \quad is_i(n, k) \in Is(n, k),$$

where  $Rs(n, k)$  and  $Is(n, k)$  are the sorted versions of the sets:

$$R(n, k) = \left\{ r_m(n, k) : Re(x(n + m)e^{-j2\pi km/N}), \right. \\ \left. m \in [-N/2, N/2) \right\}$$

$$I(n, k) = \left\{ i_m(n, k) : Im(x(n + m)e^{-j2\pi km/N}), \right. \\ \left. m \in [-N/2, N/2) \right\} \quad (4)$$

The elements:  $rs(n, k)$  and  $is(n, k)$  are sorted in non-decreasing order:  $rs_i(n, k) \leq rs_{i+1}(n, k)$  and  $is_i(n, k) \leq is_{i+1}(n, k)$ .

In order to remove the noise and to preserve good resolution, the weighting coefficients  $a_i$  are defined in analogy with the coefficients of  $\alpha$ -trimmed filter as:

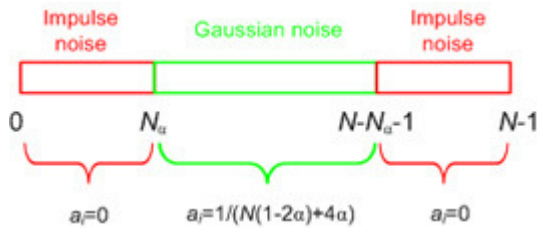


Fig. 1. The influence of parameter  $\alpha$  on noise reduction (larger  $\alpha$  results into higher reduction of impulse noise)

$$a_i = \begin{cases} \frac{1}{N(1-2\alpha)+4\alpha}, & \text{for } i \in [N_\alpha, N - N_\alpha - 1] \\ 0, & \text{elsewhere,} \end{cases} \quad (5)$$

where  $N_\alpha = (N-2)\alpha$ , while the parameter  $\alpha \in [0, 1/2]$ . The influence of  $\alpha$  on noise reduction is illustrated in Fig 1.

After the sorting operation, the coefficients corrupted by the impulse noise will be located in the regions at the beginning and at the end of the sorted sequence. Thus, in these regions the coefficients  $a_i$  should be set to zero. In order to suppress Gaussian noise, the mean value is calculated for the remaining part of the sorted sequence. Note that the larger value of  $\alpha$  provides better reduction of impulse noise, while smaller  $\alpha$  improves spectral characteristics [17]. As a special cases, for  $\alpha=0$  and  $\alpha=1/2$  the standard and median STFT forms follow, respectively.

The steps of the algorithm for realization of the L-estimate STFT are summarized in Table I.

The L-estimate STFT can be used to calculate various time-frequency distributions. For instance, the L-estimate form of the robust spectrogram can be obtained as:

$$SPEC_L(n, k) = |STFT_L(n, k)|^2 \quad (6)$$

where  $STFT_L$  denotes the L-estimate STFT. Furthermore, the L-estimate Wigner distributions is defined by using STFT as:

$$WD_L(n, k) =$$

TABLE I  
ALGORITHM FOR REALIZATION OF THE L-ESTIMATE STFT

STEP	Description
1.	Select windowed part of the signal for a current time instant $n$ : $x(n+m), m \in [-N/2, N/2]$ .
2.	Multiply the windowed signal $x(n+m)$ by the functions $e^{-j2\pi nk/N}$ for $k \in [-N/2, N/2]$ .
3.	Based on the samples amplitudes, sort the resulting sequence into a non-decreasing set.
4.	Choose the value of parametar $\alpha$ according to the noise nature (e.g. $\alpha=1/2$ for impulse noise, $\alpha=0$ for Gaussian noise, etc.)
5.	For the chosen $\alpha$ calculate the weights $a_i$ .
6.	Multiply the sorted sequence by the weights $a_i, i = 1, \dots, N$ . As a result, we obtain $STFT_L(n, k)$ for a single time instant $n$ and $k \in [-N/2, N/2]$ .
7.	Go back to step 1 and repeat the above procedure for the next time instant.

$$= \sum_{q=-N/2}^{N/2-1} STFT_L(n, k+q) \overline{STFT}_L(n, k-q), \quad (7)$$

where  $\overline{(\ )}$  denotes a conjugation operation.

### III. HARDWARE REALIZATION

Next, a hardware realization of the system for time-frequency signal analysis is considered. The general block scheme of the system for the L-estimate STFT realizations is shown in Fig 2. The input signal is multiplied by the basis functions (according to (4)) and the obtained product is further sorted within the Sorting circuit. The L-estimate STFT is obtained at the output of the L-estimate circuit as a weighted sum of the sorted elements.

The sorting operation is computationally the most demanding part of the proposed system. Various solutions for sorting analog and

digital signals have been proposed, [18]-[21]. An analog sorter proposed in [18] provides sorting operations with  $O(N)$  time complexity, while, for example, the realization of the digital Bitonic sort network has the time complexity of  $O(2\log_2 N)$  [19]. An interesting solution for a mixed analog-digital sorter with the time-complexity  $O(N)$  has been proposed in [20]. However, by increasing the number of elements, the execution time significantly increases. An analog sorter with the execution time of order  $O(1)$  has been proposed in [21]. The drawback of this system is a very complex structure that requires  $O(N^2)$  comparators and  $O(N^3)$  connection lines.

In order to provide a fast sorting operation, we propose an analog-digital hardware with execution time that does not depend on the number of input elements. The sorting operation is realized through the successive blocks named: *Comparison block*, *Combination block* and *Demultiplexers block* (Fig. 3). The input signals are compared within the *Comparison block*. The comparison results are fed to the input of *Combination block* that produces address signals for *Demultiplexers block*. Then, the *Demultiplexers block* is used to place the input signals into appropriate positions within the sorted sequence. Each of these blocks is described in more details in the sequel.

**Comparison block:** The input signals are mutually compared within *Comparison block*. The comparator has two outputs obtained simultaneously: the basic output  $C_{i,j}$  and the inverted output  $\overline{C_{i,j}}$ . Note that  $\overline{C_{i,j}} = C_{j,i}$  holds. Thus, the comparison of inputs  $V_i$  and  $V_j$  is done as follows:

$$\begin{aligned} & \text{if } i < j \text{ compare } V_i \text{ and } V_j : \\ & \{ \text{if } V_i \geq V_j \Rightarrow C_{i,j} = 1, \text{ else } C_{i,j} = 0 \}. \\ & \text{if } i > j \Rightarrow C_{i,j} = \overline{C_{j,i}}. \end{aligned}$$

For example, when  $i=1$  and  $j=2$ ,  $V_1$  and  $V_2$  are compared to obtain  $C_{1,2}$ , while  $C_{2,1}$  results as  $\overline{C_{1,2}}$ . Hence, the total number of comparators is  $N(N-1)/2$ . For the  $i$ -th input signal, the sequence  $C_{i,1}, C_{i,2}, \dots, C_{i,N}$  is obtained at the output of *Comparison block*.

**Combination block:** The sequence  $C_{i,1}, C_{i,2}, \dots, C_{i,N}$  is fed to the input of the  $i$ -

th Combination network (Fig 3). Combination network counts the number ( $K$ ) of logical values 1 within the sequence  $C_{i,1}, C_{i,2}, \dots, C_{i,N}$ . Then, it converts  $K$  into the binary sequence  $A_{i1} A_{i2} \dots A_{iP}$  ( $P=\log_2 N$ ), as:

$$K = \sum_{k=1}^P A_{i,k} 2^{k-1}.$$

The binary sequence  $A_{i1} A_{i2} \dots A_{iP}$  is the select signal for *Demultiplexers block*, i.e., it represents the position of the  $i$ -th input in the sorted sequence.

The realization of Combination network requires  $N-1$  adders. *Combination block* contains  $N$  Combination networks (one network for each input), with the total number of  $N(N-1)$  adders.

**Demultiplexers block** is realized by using  $N$  demultiplexers with  $P$  select inputs and  $N$  outputs. By using select signals  $A_{i1}, A_{i2}, \dots, A_{iP}$ , the  $i$ -th input signal  $V_i$  will be placed to the appropriate position  $V_{s_i}$  in the sorted sequence.

According to the previous analysis, the position of the  $i$ -th input signal in the sorted sequence is determined by using  $N-1$  comparators,  $N-1$  adders and a demultiplexer with  $P$  select signals and  $N$  outputs. It means that the whole system requires  $N(N-1)/2$  comparators,  $N(N-1)$  adders and  $N$  demultiplexers. The total time required to execute the sorting operation is:

$$t_u = t_{comp} + t_{cn} + t_{demux} \quad (8)$$

where  $t_{comp}$  is the comparison time,  $t_{cn}$  is the execution time of Combination network, while  $t_{demux}$  denotes demultiplexing time. Note that  $t_{comp}$  does not depend on the number of input signals, since all comparisons are performed simultaneously. The number of input signals almost does not even have an effect on the  $t_{demux}$  time. In fact, by increasing the number of input samples within the window,  $t_{cn}$  just slightly increases.

**L-estimate circuit:** The sorted sequences are further processed within the L-estimate circuit (Fig 2). It is realized by using the summer amplifier with an appropriate resistors network. The control signals  $a_i$  are used

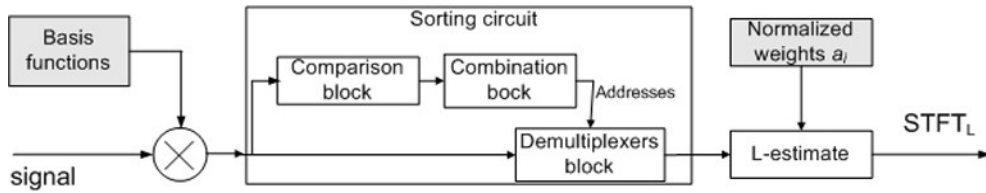


Fig. 2. A block scheme for the L-estimate STFT realization: multiplication with basis functions, sorting operations, L-estimate circuit

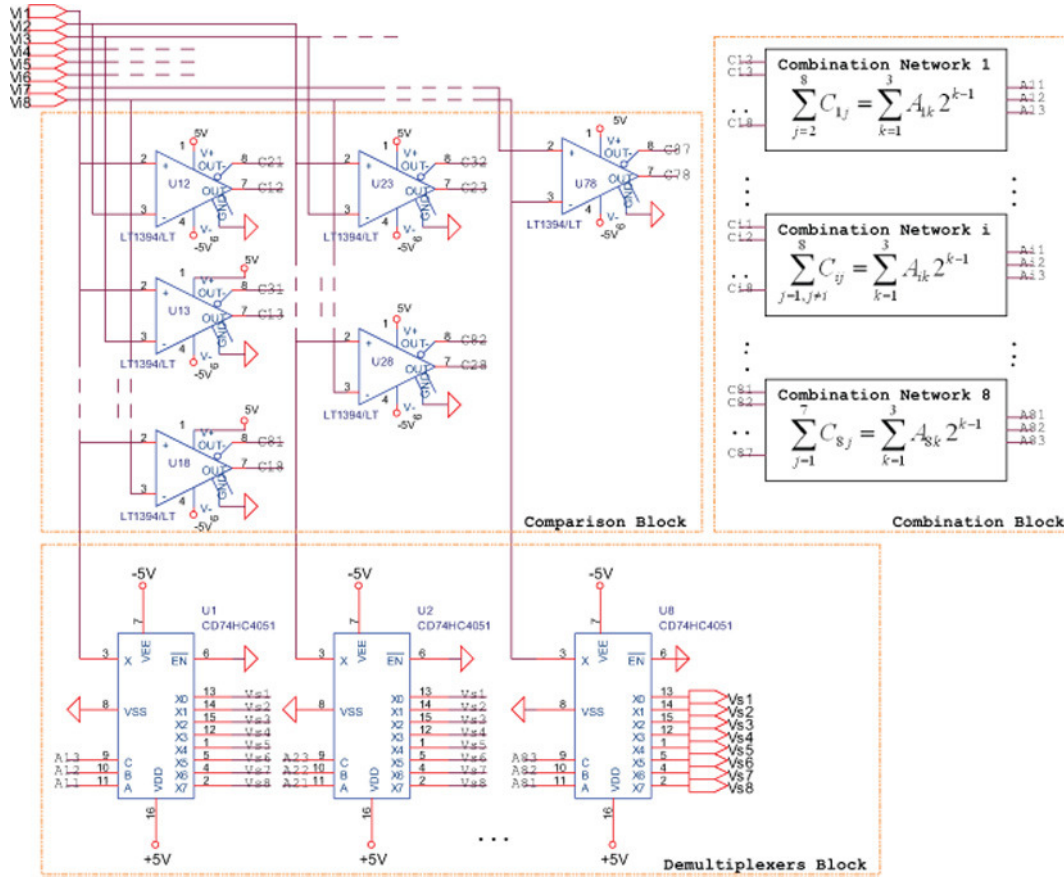


Fig. 3. An implementation of analog-digital sorter that consists of three successive blocks: Comparison block, Combination block, and Demultiplexers block

to determine which of the sorted elements will be included in the summation:

$$a_i = \begin{cases} 1, & \text{for } i \in [N_\alpha, N - N_\alpha + 1] \\ 0, & \text{elsewhere,} \end{cases}$$

while the weighting factor  $1/(N(1 - 2\alpha) + 4\alpha)$  that appears in (5) is realized by using an appropriate value of resistor  $R_1$ . An example of the L-estimate circuit with eight input signals

and  $\alpha=1/4$  is shown in Fig 4. According to (5), the coefficients  $a_1, a_2, a_7,$  and  $a_8$  are equal to zero, that means the corresponding sorted elements are not included in the summation. The value of the resistor  $R_1$  is  $R/5$ .

It is important to note that the proposed hardware can be easily adapted to various noise conditions by choosing the appropriate values for the parameter  $\alpha$  and resistor  $R_1$ .

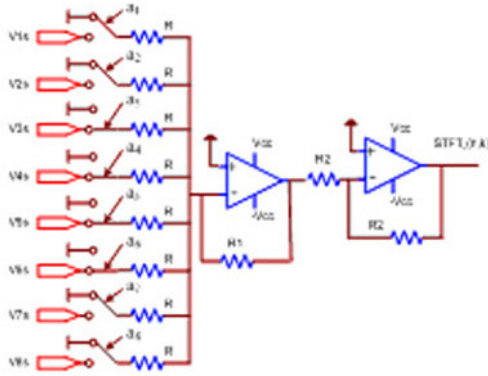


Fig. 4. A solution for the L-estimate circuit, based on a resistors network and two amplifier circuits

If the noise is Gaussian,  $\alpha=0$  and  $R_1=R/N$  should be used, resulting into the standard STFT. Similarly, for  $\alpha=1/2$  and  $R_1=R$ , the median form of STFT is obtained, which is optimal for salt and pepper impulse noise. For other types of noise the parameter  $\alpha$  should take a value between  $\alpha=0$  and  $\alpha=1/2$ , which leads to the L-estimate STFT. Thus, the resistor  $R_1$  should be:  $R_1 = R/(N(1 - 2\alpha) + 4\alpha)$ . Note that it can be realized as a voltage controlled resistor [22].

The proposed system can be easily integrated in the existing hardware solutions for realization of quadratic and higher order time-frequency distributions based on the STFT, such as the L-estimate Wigner distribution, the S-method, etc [10]-[14].

#### IV. SIMULATION AND EXPERIMENTAL RESULTS

In order to test the performance of the proposed system, its hardware realization is performed. The input signal is multiplied with the basis functions and fed to the sorter input by using D/A converter AD768. The analog-digital sorter is realized by using the LT1394 comparators, [23], CD74HC4051 demultiplexers, [24], and Quartus II for the Combination network. The L-estimate STFT is obtained at the output of the analog summing circuit that is realized by using the resistors network and LT1395 amplifiers [25]. The execution time of the proposed system is between 140 and

170 ns for signals of length 64 up to 256 time samples. Note that the comparator's execution time slightly depends on the difference between input voltages, power supply level, etc.

For the comparison, we calculated the execution time for software realization of the L-estimate STFT in Matlab7 (running on PC with 3GHz and 2GB RAM), which was found out to be around 15 ms. The realization of the L-estimate STFT based on the digital Bitonic sort algorithm, implemented in FPGA EP3SE50F780C2 device has also been considered. The execution time of this realization was between 560 and 780 ns for the signals of length 64 up to 256 time samples. Thus, the proposed analog-digital system outperforms both the software and digital hardware solution for the L-estimate STFT.

Furthermore, the proposed system has been tested in terms of setup requirements for laser welding monitoring, considered in [4]. Therein, the quadratic Margenau-Hill and Rihaczek distributions were used. For a fair comparison, as in [12], the proposed system is extended by an additional block to produce a quadratic distribution (the S-method i.e. the cross-terms free Wigner distribution). By using this hardware solution, a set of 8192 samples can be processed for approximately  $0.5 \mu s$ , so that the welding status can be monitored every 0.05 mm. These results are significantly better than the results reported in [4]. One of the reasons is that the proposed system does not require downsampling and low pass filtering, nor it requires to apply the FFT block twice, as it is the case with the Margenau-Hill and Rihaczek distributions. Note that, the robust realization of the Margenau-Hill and Rihaczek distributions can include the proposed system as an intermediate step.

Finally, we illustrate the performance of the proposed system on a real chirp signal corrupted by mixed Gaussian and impulse noise  $\xi(t) = 0.3\xi_1^3(t) + 0.3j\xi_2^3(t)$  ( $\xi_1(t)$  and  $\xi_2(t)$  are Gaussian noises with zero mean and variance equal to one). The rectangular window of width  $N=128$  is used. The execution time for the L-estimate spectrogram in this example is 157 ns. The results for the standard and the L-estimate spectrogram, obtained at the system

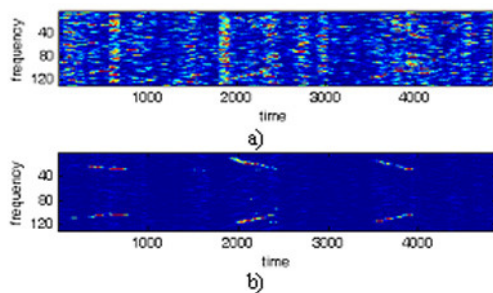


Fig. 5. Simulation results obtained with: a) Standard spectrogram b) L-estimate spectrogram

outputs, are visualized by using Matlab 7 and shown in Fig 5.a and Fig 5.b, respectively. The standard spectrogram is calculated according to (3) with  $\alpha=0$ , while for the L-estimate spectrogram the parameter  $\alpha=3/8$  is used.

The unwanted effects of impulse noise are almost completely removed when the L-estimate spectrogram is used, while the standard spectrogram does not produce satisfactory results.

#### V. CONCLUSION

A fast analog-digital system for a robust time-frequency analysis based on the L-estimate STFT has been proposed. The system is suitable for real-time applications dealing with nonstationary signals corrupted by heavy-tailed noise. As an important part of the proposed solution, the analog-digital sorter with time complexity of  $O(1)$  has been designed. It consists of  $N(N-1)/2$  comparators,  $N$  combination networks and  $N$  demultiplexers. The performance of the proposed system has been tested on a real chirp signal corrupted by mixed Gaussian and impulse noise. The system can be easily integrated into the existing solutions for realization of the L-estimate quadratic and higher order time-frequency distributions. Also, with slight modifications, the proposed system can be used for realization of robust space/spatial-frequency distributions in the case of two-dimensional signals.

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