Estimation of single-tone signal frequency by using the L-DFT

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Abstract— **Frequency estimation of complex sinusoidal signal parameters for mixed Gaussian and impulse noise environment is considered. We assume that the sinusoid has constant amplitude. The first stage in the proposed algorithm is calculation of the L-DFT forms for various parameters. Then, an optimal value of the L-DFT parameter is estimated as a value minimizing the L-DFT energy. Position of the L-DFT maximum calculated for the optimal parameter is used as a coarse frequency estimate. Fine estimation is performed by a recently proposed iterative procedure. Numerical analysis confirms accuracy of the proposed technique.**

I. Introduction

Precise estimation of sinusoidal signal frequency is an important task from both theoretical and practical point of view. Commonly, problem of precise estimation is addressed only for Gaussian noise environment. Displacement techniques based on the standard DFT are the simplest and the most efficient techniques for precise frequency estimation of these signals [1]-[4]. In numerous applications signals are corrupted by impulsive and/or heavy tailed disturbances. The standard DFT based techniques fail to produce accurate results for these environments [5], [6]. Recently, the DFT forms (robust DFTs) derived according to the robust statistics concept introduced by Huber [7] have been proposed for handling the spectral analysis issue for signals corrupted by an impulse and/or heavy-tailed noise.

The problem of precise estimation of the sinusoidal signal frequency for impulse noise environment has been addressed in [8], where the marginal-median DFT form was used as an estimation tool. This robust DFT form can be used as an estimate of the standard DFT of non-noisy signals for impulse noise environment. The coarse frequency estimation is performed by using position of the marginal-median DFT maximum. Subsequent iterations are determined according to the technique proposed by Aboutanios and Mulgrew [4]. This technique has been used in order to get fine (precise) estimate. Note that the Aboutanios and Mulgrew algorithm is proposed for Gaussian noise environment and applied to the standard DFT. The main ingredient in this procedure is the ratio of magnitudes of the DFT for two frequencies around the coarse estimate. It has been shown in [8] that the ratio of the magnitude of the marginal-median DFT for two frequencies displaced from the coarse estimate has a form similar to the standard DFT for Gaussian noise environment. Namely, this ratio has opposite sign and smaller magnitude than the displacement of the coarse estimate from the true frequency. The updated frequency is calculated as a sum of the coarse estimate and the ratio. The frequency is then updated and the procedure repeated. This technique has been shown to be convergent with the rate of convergence depending on the considered noise environment. However, in general, it is fast since the number of required iterations is relatively small.

The main drawback of the marginal-median DFT form is the spectral distortion effect [9]. It is caused by the fact that the marginal-median DFT is calculated as one (or two) modulated signal sample for each instant. In order to reduce spectral distortion effect, the L-filter DFT (L-DFT) forms are proposed in [9]. The L-DFT can be used for spectral analysis of signals corrupted by mixed Gaussian and impulse noise.

In this paper we propose a precise estimation of the frequency of sinusoids corrupted by mixture of Gaussian and impulse noise by using the L-DFT [9]. In the first stage, a simple strategy for determination of the sub-optimal

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L-DFT is applied. The coarse estimation of the signal frequency is performed by using position of the L-DFT maximum. The fine estimation of displacement from the frequency grid is performed by an iterative procedure from the Aboutanios and Mulgrew algorithm [4]. In this paper, the iterative procedure is applied to the L-DFT for signals corrupted by mixture of Gaussian and impulse noise.

The paper is organized as follows. A brief overview of the robust DFT forms is given in Section 2. Determination of the optimal robust DFT form is considered in Section 3. The proposed frequency estimation technique is summarized in Section 4. Results of experiments are presented in Section 5.

II. Robust DFT forms

In this paper we consider the complex sinusoid with constant amplitude, $f(n)= A\exp(j\mathrm{w}_0 n+j\varphi)$, embedded in the white noise $x(n)=f(n)+\mathrm{n}(n)$. Our goal is to perform estimation of the frequency of the sinusoid based noisy observations since it is crucial parameter of these signals. For Gaussian noise environment the basic tool for frequency estimation is the standard DFT that can be defined as:

$$X_S(k)=\frac{1}{N}\sum_{n=-N/2}^{N/2-1}x(n)\exp(-j2\pi nk/N)=$$

$$=\mathrm{mean}\ \{\mathrm{x(n)}\exp(-\mathrm{j}2\pi\mathrm{nk/N})\ |n\in[-N/2,N/2)\} \quad (1)$$

for $k\in[-N/2,N/2)$. This DFT form is the ML estimate of the signal spectra for Gaussian noise environment where $\mathrm{n}(n)$ is a white complex-valued Gaussian noise with independent real and imaginary parts. The standard DFT can be calculated as a solution of the following optimization problem (common for ML estimates):

$$X_S(k)=$$

$$=\arg\min_{\mu}\sum_{n=-N/2}^{N/2-1}F(|x(n)\exp(-j2\pi nk/N)-\mu|), \quad (2)$$

where the loss function is $F(\mathrm{x})=|\mathrm{x}|^2$. However, the standard DFT, like other linear techniques, exhibits weak performance for signals corrupted by impulse noise. For a known noise environment, the corresponding ML DFT form can be determined. In particular, the ML DFT form for Laplacian noise with independent real and imaginary parts is obtained with the loss function $F(\mathrm{x})=|\mathrm{Re(x)}|+|\mathrm{Im(x)}|$, leading to the marginal-median form of the robust DFT [6]:

$$\begin{aligned}X_M(k)=&\ \mathrm{median}\,\{\mathrm{Re[x(n)}\exp(-\mathrm{j}2\pi\mathrm{nk/N})],\ n\in[-N/2,N/2)\}\\ &+j\mathrm{median}\,\{\mathrm{Im[x(n)}\exp(-\mathrm{j}2\pi\mathrm{nk/N})],\ n\in[-N/2,N/2)\}\end{aligned} \quad (3)$$

For a white a-stable noise environment, the myriad filter forms are proposed with the loss function $F(\mathrm{x})=\log(|\mathrm{x}|^2+K^2)$, where K is the so-called linearization parameter [9]-[11].

These ML forms can be sensitive to variations in the assumed model of noise environment. For this reason the L-DFT forms were proposed in [9]. They are very effective in a mixed Gaussian and impulse noise environment. Also, L-DFT forms introduce smaller spectral distortion effects than the marginal-median DFT. The L-DFT can be defined as:

$$X_L(k)=\sum_{l=0}^{N-1}a_l[r_l(k)+ji_l(k)], \quad (4)$$

where a_l, l=0, 1, ..., N-1 are L-estimate coefficients, while $\mathbf{r}_l(k)$ and $\mathbf{i}_l(k)$ are ordered elements from the sets: $\mathbf{r}_l(k)\in\mathbf{R}(k)=\{\mathrm{Re}\{x(n)\exp(-j2\pi nk/N)|n\in[-N/2,N/2)\}$ and $\mathbf{i}_l(k)\in\mathbf{I}(k)=\{\mathrm{Im}\{x(n)\exp(-j2\pi nk/N)|n\in[-N/2,N/2)\}$, with $\mathbf{r}_l(k)=\mathbf{r}_{l+1}(k)$ and $\mathbf{i}_l(k)=\mathbf{i}_{l+1}(k)$. Coefficients of the L-filter are commonly selected as $\sum_{l=0}^{N-1}a_l=1$ and $a_l=a_{N-1-l}$. Due to its simplicity, we will consider the a-trimmed mean form of the L-DFT with coefficients determined as (under the assumption that N is even):

$$a_l=\begin{cases}\frac{1}{N(1-2\alpha)} & l\in[\alpha N,N(1-\alpha)-1]\\ 0 & \text{elsewhere.}\end{cases} \quad (5)$$

Special cases of the L-DFT are the standard DFT for a=0 (for $\forall a_l=1/N$) and the marginal-median DFT form [6] for a=0.5-2/N ($a_{N/2-1}=a_{N/2}=1/2$ and $a_i=0$ elsewhere).

These two special L-DFT forms exhibit quite different behavior: the standard DFT produces the ML estimate for Gaussian noise environment, but is sensitive to even small amount of impulse noise; the marginal-median DFT is robust to impulse noise influence but can introduce the spectral distortion effect [6], [9]. It suggests that it is possible to design optimal L-DFT form for particular signal and noise environment by selecting the value of parameter a. This form should produce a trade-off between robustness to the noise influence and spectral distortion effects. Several techniques for adaptive determination of the parameter a in the L-DFT are reviewed in [9], [13]. Alternative techniques can be found in [14]-[16]. In addition, determination of the adaptive K parameter in the myriad form of the robust DFT is considered in [10], [11]. These methods are auxiliary tools for our research. In the next section, a technique developed in [9] that can be applied for FM signals with constant amplitude embedded in an impulse noise, will be described.

III. Optimal L-DFT

Optimal selection of the parameter a in the a-trimmed mean DFT is still an active research topic. However, under the assumption that signal of interest is pure sinusoid with constant amplitude and that we have relatively large number of signal samples, a very simple procedure for adaptive selection of a can be utilized (already used in [9]). Denote the a-trimmed mean DFT with particular parameter a as $X_a(k)$. This $X_a(k)$ can be written as:

$$X_a(k) \approx F(k) + N_a(k) + D_a(k), \qquad (6)$$

where $F(k)$ is the standard DFT of non-noisy signal, while $N_a(k)$ is the residual noise term and $D_a(k)$ is spectral distortion term. Under the assumption that $F(k)$, $N_a(k)$ and $D_a(k)$ are mutually independent, one can use the following approximate expression for energy of the $X_a(k)$:

$$\sum_{k=-N/2}^{N/2-1} |X_\alpha(k)|^2 \approx$$

$$\approx \sum_{k=-N/2}^{N/2-1} \{|F(k)|^2+|N_\alpha(k)|^2+|D_\alpha(k)|^2\}. \qquad (7)$$

This conclusion has been confirmed by simulations for the considered type of signals embedded in different noise environments [9]. Under introduced realistic assumptions for the considered signal type, parameter a that produces the smallest energy of transform $\sum_{k=-N/2}^{N/2-1} |X_\alpha(k)|^2$ gives the smallest joint influence of the noise and spectral distortion effects:

$$\hat{\alpha}_{opt} = \arg \min_{\alpha\in[0,0.5]} \sum_{k=-N/2}^{N/2-1} |X_\alpha(k)|^2. \qquad (8)$$

Note that a similar methodology can be used for determination of the optimal parameter K if the myriad DFT forms are used for estimation of signals with constant amplitude. Some other sophisticated statistical techniques for selection of the optimal parameters for FM signals with time-varying amplitude are reviewed in [11].

IV. Frequency estimation

Assume that complex sinusoidal $f(t)=A\exp(j\mathrm{w}_0 t+j\varphi)$ embedded in a white noise environment, $x(t)=f(t)+\mathrm{n}(t)$, is considered within $t\in$[-$T/2,T/2$) and sampled with D$t=T/N$, i.e., $x(n)=x(n\mathrm{D}t)$, $n\in$[-$N/2$,$N/2$).

The optimal L-DFT obtained by the procedure described in Section 3, $X_L(k) = X_{\hat{\alpha}_{opt}}(k)$, or the L-DFT with fixed parameter a, $X_L(k)=X_a(k)$, can be used as a coarse frequency estimator:

$$\hat{\omega} = \hat{k}_0 \Delta\omega \qquad \hat{k}_0 = \arg\max_k |X_L(k)| \qquad (9)$$

with Dw=$2\pi/T$. In order to handle the issue of precise frequency estimation, we adopted the iterative technique proposed in [4] for the standard DFT and for signals corrupted by

Gaussian noise. This algorithm can be summarized as follows.

Step 1. Calculation of the robust DFT (4) and (5) and determination of the optimal L-DFT by using the procedure given in Section 3. Estimate $\hat{k}_0$ as

$$\hat{k}_0 = \arg\max_k |X_L(k)|. \tag{10}$$

Step 2. Set $\hat{\rho}_0 = 0$ and i=0.

Step 3. Calculate

$$\hat{X}_g = \sum_{l=0}^{N-1} a_l[r_l(\hat{k}_0 + \hat{\rho}_i + g) + ji_l(\hat{k}_0 + \hat{\rho}_i + g)],$$
$$forg = \pm 0.5. \tag{11}$$

Step 4. Next iterations are evaluated as [4]:

$$\hat{\rho}_{i+1} = \hat{\rho}_i - h(\rho_i) \tag{12}$$

where

$$h(\hat{\rho}_i) = \frac{1}{2}\frac{|\hat{X}_{0.5}| - |\hat{X}_{-0.5}|}{|\hat{X}_{0.5}| + |\hat{X}_{-0.5}|}. \tag{13}$$

Set i=i+1, and repeat steps 3 and 4.

Step 5. After a specific number of iterations Q, the frequency of sinusoid is estimated as:

$$\hat{\omega} = (\hat{k}_0 + \hat{\rho}_Q)\Delta\omega. \tag{14}$$

Comments on the algorithm

Note that an alternative form of update rule (13) has been proposed in [4]. However, this alternative form does not produce accurate results for the robust DFTs and we decided to use (13) that is accurate for the considered signal and noise model.

Note that $\mathbf{r}_l(\hat{k}_0 + \hat{\rho}_i + g)$ and $\mathbf{i}_l(\hat{k}_0 + \hat{\rho}_i + g)$ in (11) are sorted elements from the sets $\mathbf{r}_l(\hat{k}_0 + \hat{\rho}_i + g) \in \mathbf{R}(\hat{k}_0 + \hat{\rho}_i + g)$=$\{\mathrm{Re}[x(n)\exp(-j2\pi n(\hat{k}_0 + \hat{\rho}_i + g)/N)] | n \in [-N/2, N/2)\}$ and $\mathbf{i}_l(\hat{k}_0 + \hat{\rho}_i + g) \in \mathbf{I}(\hat{k}_0 + \hat{\rho}_i + g)$= $\{\mathrm{Im}[x(n)\exp(-j2\pi n(\hat{k}_0 + \hat{\rho}_i + g)/N)] | n \in [-N/2, N/2)\}$, with $\mathbf{r}_l(\hat{k}_0+\hat{\rho}_i+g)$=$\mathbf{r}_{l+1}(\hat{k}_0+\hat{\rho}_i+g)$ and $\mathbf{i}_l(\hat{k}_0+\hat{\rho}_i+g)$=$\mathbf{i}_{l+1}(\hat{k}_0+\hat{\rho}_i+g)$. Samples $\hat{X}_{\pm 0.5}$ are dislocated from the detected maximum in previous phase for a half of the frequency sampling interval ±Dw/2. Coefficients a_l are given by (5) where parameter a can be fixed or determined by using the proposed procedure (8).

This procedure requires evaluation of the L-DFT for N+2Q frequencies. Note that, in our simulation, the required number of iterations is below Q=8, i.e., $Q \ll N$. This procedure, applied on the standard DFT, produces results very close to the Cramer-Rao lower bound within only Q=2 iterations [4]. In [8] this procedure has been applied for the marginal-median DFT. It has been shown that convergence is stable for various noise environments and that accurate results are achieved for Q∈[2,7]. Improvement achieved by further increase of Q is negligible. Thus, this procedure is more efficient than the frequency interpolation by using the zero padding. Namely, the zero padding would require evaluation of the L-DFT for RN frequencies where, typically, $R \gg 1$ and (N+2Q) ≪ RN.

V. NUMERICAL ANALYSIS

In the experiment we consider a single-tone sinusoidal signal:

$$f(t) = \exp(j\omega_0 t + j\delta t + j\varphi), \tag{15}$$

within t∈[-T/2,T/2) with T=2. Experiments have been performed with various numbers of samples and here we present results obtained with N=32, 256 and 1024. In the simulations, we set w$_0$=2$\pi k_0/T$, where k_0=12, and in each trial of the Monte Carlo simulations, d and φ have been selected randomly with uniform distribution on the intervals d∈[-π/T,π/T] and φ∈[-π/2,π/2], respectively. Signal has been embedded in mixed Gaussian and impulse noise:

$$x(t) = f(t) + n_G(t) + n_I(t), \tag{16}$$

where n$_G(t)$ is a white complex Gaussian noise with variance s^2, while n$_I(t)$ is a model of impulse noise. Impulses appear with probability p in both real and imaginary parts. We assume that negative and positive impulses appear with the same probability $p/2$, with a constant amplitude a=5.

We compared the method based on the standard DFT [4] with the proposed technique based on the optimal L-DFT, L-DFT with

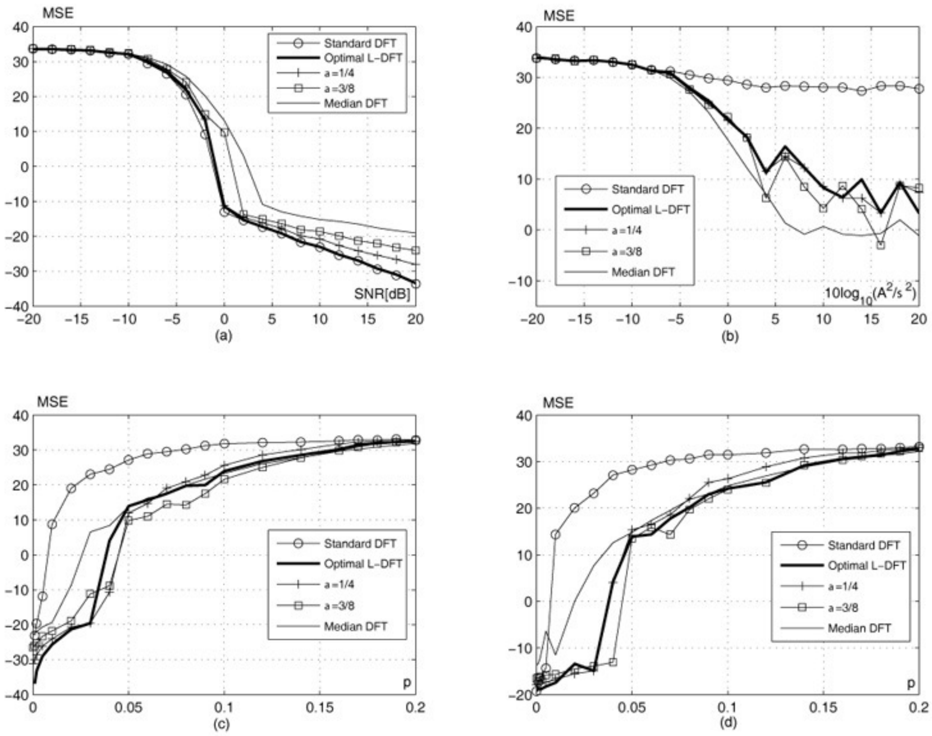

Fig. 1. Mean squared error in frequency estimation for N=32 samples within $t\in[-T/2,T/2)$, T=2: (a) Pure Gaussian noise as a function of SNR; (b) Mixed Gaussian and impulse noise for fixed percentage of impulses 5% as a function of ratio of signal power and variance of Gaussian noise; (c) Pure impulse noise as a function of percentage of noise; (d) Mixed Gaussian and impulse noise for fixed variance of the Gaussian noise σ^2=0.25 as a function of percentage of impulse noise.

fixed value of the parameter a=1/4 and a=3/8 and the procedure applied to the marginal-median DFT form [8]. The mean squared error (MSE) in dB defined as:

$$MSE = 10\log_{10} E\{[\hat{\omega} - (\omega_0 + \delta)]^2\} \qquad (17)$$

is depicted in Figs. 1-3 for N=32, N=256 and N=1024, respectively. The following noise environments are considered: (a) Pure Gaussian noise as a function of SNR; (b) Mixed Gaussian and impulse noise for fixed amount of impulse noise p=5%, as a function of signal to Gaussian noise ratio; (c) Pure impulse noise as a function of p; (d) Mixed Gaussian and impulse noise for fixed amount of Gaussian noise s^2=0.25 as a function of p. In all experiments, the number of iterations in the proposed procedure was Q=5.

For pure Gaussian noise (Figs.1a, 2a and 3a), the proposed technique behaves the same as the technique from [4] applied to the standard DFT. Note that the technique from [4] produces excellent accuracy for Gaussian noise environment that is just 1.5% above the Cramer Rao lower bound. Results obtained in this case confirm that the proposed technique for determination of the optimal a-trimmed mean DFT selects the standard DFT as an optimum for this environment. For small number of samples, N=32, and other noise environments (Figs.1b, c, d), it can be seen that the L-DFT forms with fixed a outperform the L-DFT with parameter a calculated by using (8). The reason is in fact that we need relatively large number of samples in order that the residual noise influence and distortion effects could be assumed independent. However, for larger number of samples (Figs. 2 and 3) the considered technique for determina-

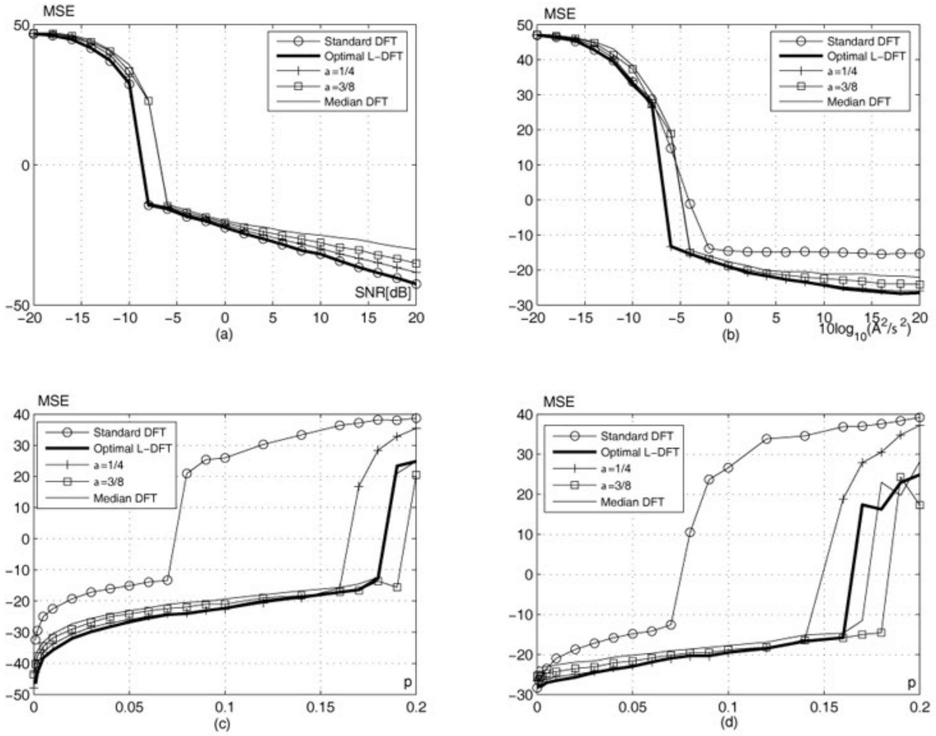

Fig. 2. Mean squared error in frequency estimation for N=256 samples within $t\in[-T/2,T/2)$, T=2: (a) Pure Gaussian noise as a function of SNR; (b) Mixed Gaussian and impulse noise for fixed percentage of impulses 5% as a function of ratio of signal power and variance of Gaussian noise; (c) Pure impulse noise as a function of percentage of noise; (d) Mixed Gaussian and impulse noise for fixed variance of the Gaussian noise σ^2=0.25 as a function of percentage of impulse noise.

tion of adaptive a produces very accurate results. For the second noise environment with fixed percentage of the impulse noise p=5% (Figs.2b and 3b), it can be seen that for small amount of Gaussian noise, the new technique outperforms the standard one by more than 10dB, but that with increase of the Gaussian noise influence this difference decreases. For the first two noise environments, the common detection threshold effect can be noticed for small SNR [12]. In the case of the pure impulse noise shown in Figs.2c and 3c, the proposed technique performs significantly better (by more than 15dB) than the standard technique for a wide range of p values. This improvement exists even for a small amount of impulse noise (for example p=0.2%). Finally, for the second considered form of the mixed noise with fixed amount of Gaussian noise, we can see that again the standard DFT based technique produces results similar to the proposed technique only for a very small amount of impulse noise. The marginal-median DFT form performs worse than the other considered algorithms for the Gaussian noise environment. However, for other three considered noise environments it performs between the standard DFT and the proposed procedure. As mentioned before, the marginal-median DFT introduces the spectral distortion effects [6], [9]. These effects are the reason why the L-DFT procedure with optimal a outperforms the marginal-median DFT. The L-DFT forms with fixed a=1/4 and a=3/8 produce results that are close to the optimal one for almost all considered noise environments.

In addition, we considered behavior of the MSE for fixed d within Monte Carlo simulation. Again, in each trial we selected randomly $\varphi\in[-\pi/2,\pi/2]$. Obtained MSE for optimal L-

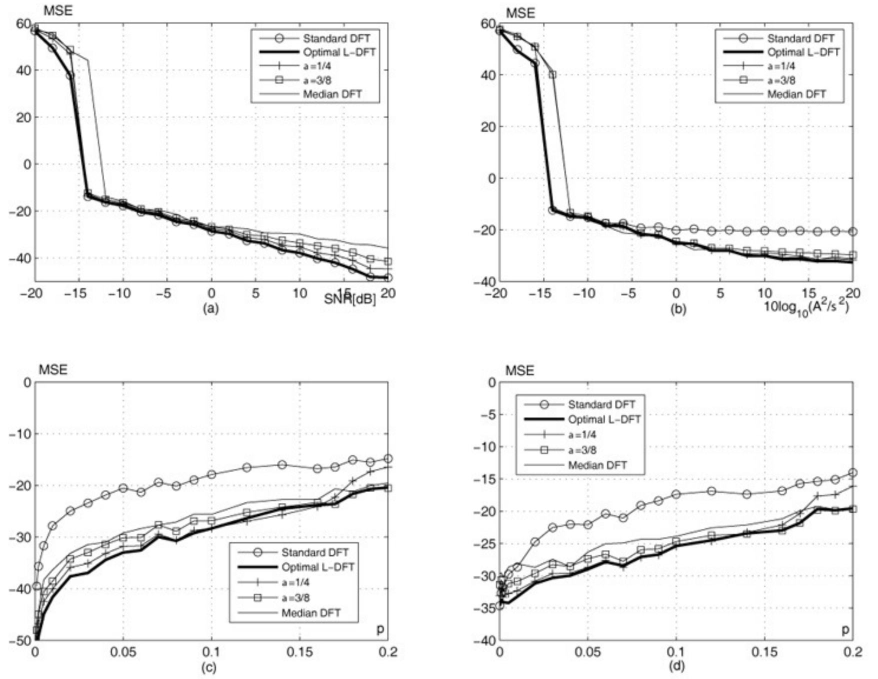

Fig. 3. Mean squared error in frequency estimation for N=1024 samples within $t\in[-T/2,T/2)$, T=2: (a) Pure Gaussian noise as a function of SNR; (b) Mixed Gaussian and impulse noise for fixed percentage of impulses 5% as a function of ratio of signal power and variance of Gaussian noise; (c) Pure impulse noise as a function of percentage of noise; (d) Mixed Gaussian and impulse noise for fixed variance of the Gaussian noise σ^2=0.25 as a function of percentage of impulse noise.

DFT for six different noise environments for N=1024 with respect to d$\in(0,\pi/T]$ is depicted in Fig. 4. The MSE for a coarse estimate (9) is given with dashed line, after one iteration with dotted line and after 5 iterations with solid line. It can be seen that obtained accuracy after just 5 iterations is approximately constant for the considered noise environments.

VI. CONCLUSION

An effective technique for estimation of single-tone sinusoidal signal frequency is proposed. The technique is based on three ingredients: the robust DFT form (here the a-trimmed mean DFT is considered); the optimization technique that determines parameter a in the robust DFT used as the coarse frequency estimate; the iterative approach recently proposed by Aboutanios and Mulgrew for obtaining precise frequency estimate. Accuracy of the proposed technique has been tested in the numerical study and compared with the original approach and with the iterative approach applied to the marginal-median DFT. For Gaussian noise environment the proposed technique behaves like the technique developed for the standard DFT, while for the impulse and mixed impulse and Gaussian noise environment the proposed technique outperforms both the standard and marginal-median DFT based algorithms. In future research we will try to determine asymptotic accuracy of the proposed procedure, to consider its application for the myriad DFT forms, to employ some more sophisticated schemes for optimal DFT determination and, finally, to apply the procedure for signals with multiple tones.

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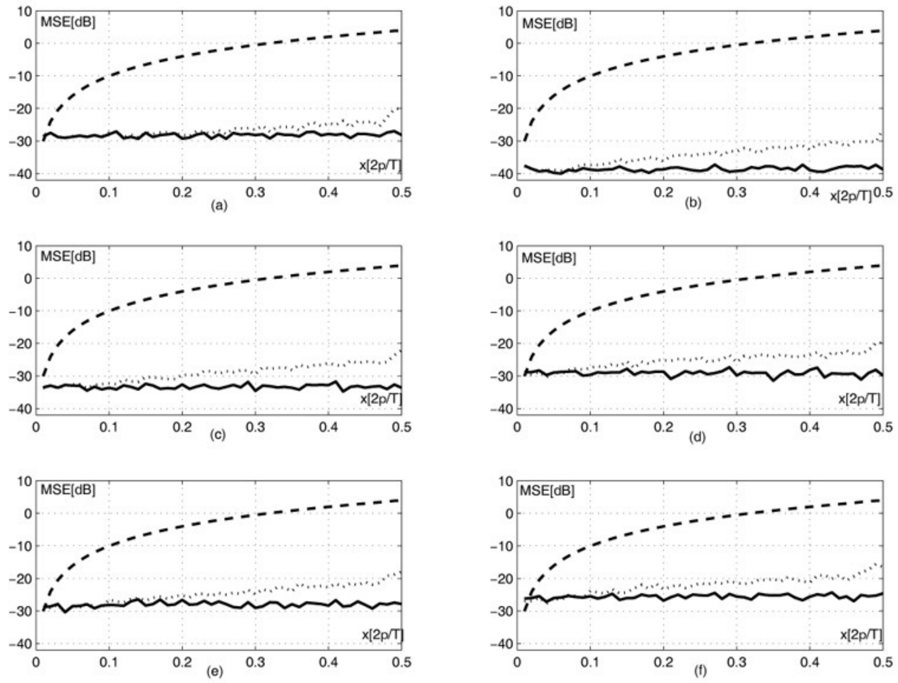

Fig. 4. Mean squared error in frequency estimation as function of d for optimal L-DFT for N=1024 samples: Dashed line – Coarse estimation (9); Dotted line – After one iteration; Solid line – After 5 iterations. (a) Pure Gaussian noise with s^2=1; (b) Pure impulse noise with p=0.02; (c) Pure impulse noise with p=0.05; (d) Mixed noise with p=0.05 and s^2=0.25; (e) Pure impulse noise with p=0.1; (f) Mixed noise with p=0.1 and σ^2=0.25.

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