Analysis of meridian estimator performance for non-Gaussian PDF data samples

Dmitry A. Kurkin, Alexey A. Roenko, Vladimir V. Lukin, and Igor Djurović

Abstract— **A sample meridian estimator of location parameter (LP) has been proposed recently and shown to be robust and controllable by means of a tunable parameter δ. The estimator properties have been initially studied but not analyzed thoroughly. In this paper we address several practical questions. First, we analyze conditions under which statistical properties of meridian estimator of LP considerably differ from those ones of the sample median estimator. Second, we give examples of probability density functions for which the sample meridian estimates can be sufficiently more accurate than sample median. Third, we consider practical situations when useful properties of the meridian estimator of LP can be exploited in practice of signal and data processing.**

I. Introduction

During many years, Gaussian model of noise and measurement errors has been widely used. However, starting from the end of sixtieth of the previous century, non-Gaussian models have attracted wide attention due to the fact that in numerous applications they describe many natural phenomena more adequately [1]. Due to this, it became necessary to consider robustness aspects while designing methods for data, signal and image processing [2, 3]. Different methods of robust estimation were put into basis of filters [3, 4] and other means of data processing intended for spectral analysis [5], noise variance estimation [6], etc. Some of them [4, 6-8] use robust estimators, in particular, a sample myriad estimator [4, 6, 8] with adjustable parameters in order to provide desirable properties for a given application.

Recently a new, sample meridian estimator has been proposed [9]. It has some similarities to both sample median and myriad estimators.

Telecommunications and Radio Engineering, Vol. 69, No. 8, Avg. 2010.

Similarly to the myriad estimator, the sample meridian estimator has a tunable parameter (δ) that controls its performance. For small δ, the sample meridian tends to be a distribution mode finder [9]. At the same time, for relatively large δ the properties of a sample meridian practically coincide with the properties of the sample median estimator. Moreover, a sample meridian is always an element of a given data sample.

However, this is a very general description of the meridian estimator. Any robust estimator is of practical interest if it differs from other ones by some peculiarities (features) that can be useful for, at least, several applications (practical situations). Such positive features could be better robustness to outliers, lower computational complexity, higher accuracy of sample location or scale estimation, optimality or quasi-optimality for some distribution or a family of probability density functions (PDFs), simplicity of parameter adaptation in comparison to other known estimators, etc. From this viewpoint, properties of any recently developed estimator should be thoroughly studied before finding proper applications for this estimator.

Although good theoretical analysis of the meridian estimator has been carried out in [9], quite many questions remained unclear (recall how many efforts were spent on analysis of the median estimate for various applications, see [10] and references therein). In particular, it is desirable to know when sample median and meridian coincide and when they considerably differ from each other. It is possible to expect that this can depend upon sample size, underlying distribution and parameter δ. When sample median and meridian do not coincide, it is desirable to determine for what situations the meridian estimator is able to produce some benefits, e.g., better accuracy of distribution parameter estimation. After this, one can expect that obtained information will clarify applications for which the meridian estimator could be a proper choice. Thus, the goal of this paper is to produce additional insights on properties of the meridian estimator and to give some practical recommendations on setting parameter δ.

II. Meridian estimator: definition and basic properties

Analytically the sample meridian estimator of location $\hat{\beta}$ is defined as

$$\hat{\beta} = \arg\min_{\beta} \left[\sum_{i=1}^{N} \ln\left(\delta + |x_i - \beta|\right) \right] = \\ = \mathrm{meridian}\{\mathrm{x_i},\ \mathrm{i} = 1, ..., \mathrm{N};\ \delta\} \qquad (1)$$

where N denotes a sample size; x_i is an i–th element of the sample; δ is also called medianity parameter [9].

Let us briefly remind basic properties of meridian estimate [9]. Its cost function is

$$\varphi\left(\beta\right) = \sum_{i=1}^{N} \ln\left(\delta + |x_i - \beta|\right). \qquad (2)$$

The function $\varphi\left(\beta\right)$ monotonically decreases for $\beta < x_{\min}$ and increases for $\beta > x_{\max}$ where $x_{\min}$ and $x_{\max}$ are minimal and maximal elements of a data sample, respectively. Thus, minima of cost function are from $x_{\min}$ to $x_{\max}$ and the number of minima is limited, it increases if δ reduces. Moreover, minima can be observed only for β that coincides with one of elements of original sample. These properties show that a very simple algorithm for finding $\hat{\beta}$ can be realized. One has to calculate $\varphi\left(x_i\right),\ i = 1, ..., N$ and to find such i for which $\varphi\left(x_i\right)$ is the smallest. As it is seen, the algorithm is considerably simpler and faster than for the myriad estimate [4]. Moreover, in opposite to sample median, sorting is not required for finding sample meridian.

Note that the meridian estimator is invariant to translation of distribution center (LP or mean if it exists). This allows restricting consideration of meridian estimator properties by the case of distributions with zero mean (center). Besides, below we will concentrate on studying only distributions symmetric with respect to their means. It is proven in [9] that for such distributions the meridian estimator produces unbiased estimates for arbitrary δ.

Certainly, special attention in our analysis should be paid to non-Gaussian heavy tailed distributions. Thus, let us consider the following seven PDFs. The first one is Gaussian PDF with variance σ_G^2 and the second PDF is Cauchy one which is a particular case of symmetric α-stable distributions: $f_1(\gamma; x) = \frac{\gamma}{\pi(\gamma^2 + x^2)}$ where γ is the parameter characterizing PDF scale. For both PDFs, their maxima are bell-shaped and continuous derivatives. We have also analyzed two particular cases of α-stable distributions (α=1.5 that has lighter tail than the Cauchy PDF and α=0.7 that is characterized by very heavy tail).

Besides, let us analyze PDFs $f_2(x)$, $f_3(x)$, $f_4(x)$ of the following three random variables:
1) $Y_1 = X_1 X_2$ (denoted as dgauss);
2) $Y_2 = X_1 X_2 X_3$ (denoted as tgauss);
3) $Y_3 = (X_1)^3$ (further referred as gauss3).

Here X_1, X_2, and X_3 are independent zero mean Gaussian variables with standard deviations σ_{X1}, σ_{X2}, σ_{X3}, respectively. All three PDFs $f_2(x)$, $f_3(x)$, $f_4(x)$ have heavier tails than Gaussian. If $\sigma_G = \sigma_{X1} = \sigma_{X2} = \sigma_{X3} = 1$, then for the PDFs $f_2(x)$ and $f_3(x)$ their variances are equal to 1.0 and for the PDF $f_4(x)$ the variance is approximately equal to 15.0. One more peculiarity of PDFs $f_2(x)$, $f_3(x)$, $f_4(x)$ is that they all have peaky (sharp, not bell-shaped) maxima [11], [12]. Random variables with PDFs $f_2(x)$ and $f_3(x)$ take place in bispectral signal processing [11], PDF $f_4(x)$ has been considered in the paper [12]. Since PDFs $f_2(x)$, $f_3(x)$, $f_4(x)$ have heavy tails, it is reasonable to characterize their scale by a robust estimate of scale. Median of absolute deviations (MAD) [3] can serve as such a characteristic. For PDF $f_2(x)$, $MAD = 0.545\sigma_{X1}\sigma_{X2}$; for $f_3(x)$ one has $MAD = 0.292\sigma_{X1}\sigma_{X2}\sigma_{X3}$; for $f_4(x)$ $MAD = 0.462\sigma_{X1}^3$. Recall that the following expressions are valid for Gaussian and Cauchy PDFs: $MAD = \sigma_G/1.483$ [3] and $MAD = 1.5\gamma$[13].

A characterization of tail weight could be percentile coefficient of kurtosis (PCK) [14]. For Gaussian PDF it is equal to 0.26 [13], for heavier tail distributions the PCK values are smaller: 0.235 for α-stable PDF with α=1.5, 0.16 for Cauchy PDF, 0.088 for α-stable PDF with α=0.7; 0.178, 0.132, and 0.076 for PDFs $f_2(x)$, $f_3(x)$, and $f_4(x)$, respectively. Note that it is shown in [15] that if a random variable is a product of two or more random variables, then tail weight for this "product" random variable is larger than tail weight of any component random variable. This explains why PDFs $f_2(x)$, $f_3(x)$, and $f_4(x)$ are heavy tailed.

The first step in analysis of the sample meridian estimator was to get imagination about conditions under which a sample meridian differs from median for the same data sample. For this purpose, let us determine probability P that meridian$\{\mathrm{x_i},\ \mathrm{i} = 1, ..., \mathrm{N};\ \delta\}$ = median$\{\mathrm{x_i},\ \mathrm{i} = 1, ..., \mathrm{N}\}$. A preliminary analysis has demonstrated that for N=3 the probability is equal to unity for all seven considered PDFs and for any δ. So, let us study larger N, namely N=5 and 11 that correspond to typical situations in signal processing in a sliding window manner [3]. The obtained dependencies are presented in Fig. 1. For the PDFs $f_2(x)$, $f_3(x)$, $f_4(x)$ we used $\sigma_{X1} = \sigma_{X2} = \sigma_{X3} = 1$, for Gaussian distribution $\sigma_G = 1$, for Cauchy PDF γ is equal to 1.

As it is seen, all of them have similar behavior in the sense that they have "saturation" to unity for rather large δ and a flat region with $P \neq 1$ for rather small values of δ. Transition zones between these two "saturations" are observed for δ within the limits from 10^{-2} to 10^2 or, more generally speaking, for δ/MAD from about 10^{-2} to 10^2. Note that for heavy tailed distributions it is more correct to analyze the obtained dependencies with respect to normalized values δ/MAD since standard deviation is a non-robust characteristic (estimate) of scale and theoretically it can be infinite (not defined) as for the family of symmetric α-stable distributions.

At the same time, there are some differences and other interesting observations. First, P becomes practically equal to 1.0 for δ exceeding 10 standard deviations of Gaussian noise (σ_G). Moreover, for sharp peak PDFs, P reaches 1.0 for δ/MAD of the order 10…50.

The second observation that follows from comparison of plots in different Figures 1,a-d is that the probabilities P depend upon N for small δ. For larger N, P becomes smaller. The smallest P takes place for Gaussian PDF, for other distributions that are heavy tailed the values of P are approximately the same.

Dependences of P on δ for N=5 (a) and N=11 (c) for Gaussian PDF and Cauchy PDFs, and for N=5 (b) and N=11 (d) for PDFs $f_2(x)$, $f_3(x)$, and $f_4(x)$

The found tendencies simplify further analysis of statistical characteristics of the meridian estimator. Obviously, there is no sense to study them for δ/MAD larger than 100 since then the sample meridian and median coincide and, thus, have the same statistical characteristics. Besides, the properties of the sample meridian and median differ in the area of δ/MAD<100 although this does not necessarily mean that in this case the meridian estimator accuracy is better than that of the median estimator.

III. Analysis of statistical characteristics of sample meridian estimator

For analysis of meridian estimator accuracy, we have determined root mean square error (RMSE) σ_β of the obtained estimates. Besides, we used MAD_β = median$\{\left|\hat{\beta}_\mathrm{j} - \mathrm{median}\{\hat{\beta}_\mathrm{j}\}\right|,\ \mathrm{j} = 1, ..., \mathrm{N_{exp}}\}$ where $\hat{\beta}_j$ is the meridian estimate for the j-th sample of data obeying a given PDF, N_{exp} defines the number of experiments (analyzed data samples, N_{exp} = 1000). Expedience of analyzing MAD_β stems from the fact that has been found in experiments – PDF of the obtained meridian estimates of location for PDFs $f_2(x)$, $f_3(x)$, and $f_4(x)$ occurred to be symmetric but non-Gaussian. The same holds for α-stable distribution with α=0.7. Similar effects have been observed for sample median estimates in the case of Laplacian distribution [14].

For Gaussian and Cauchy distributions,

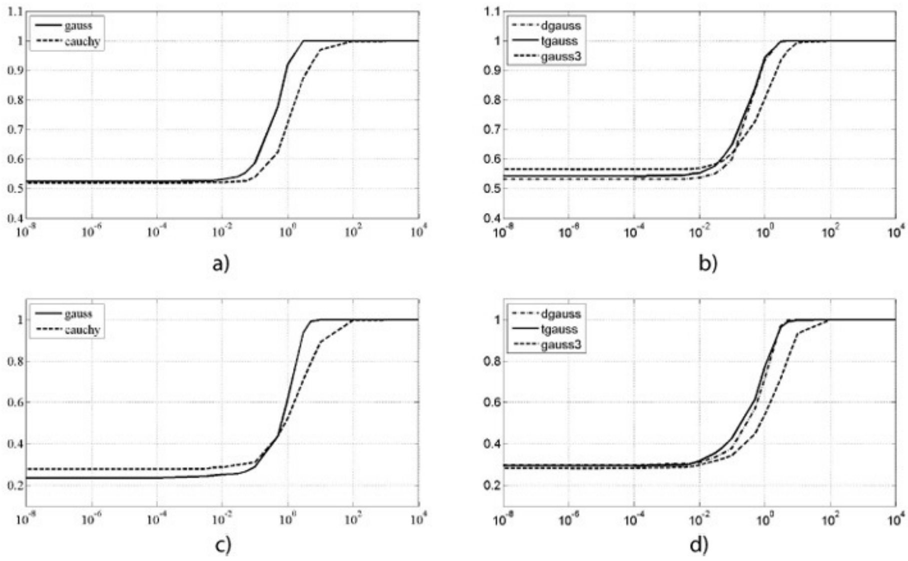

Fig. 1. Dependences of P on δ for N=5 (a) and N=11 (c) for Gaussian PDF and Cauchy PDFs, and for N=5 (b) and N=11 (d) for PDFs $f_2(x)$, $f_3(x)$, and $f_4(x)$

the meridian estimate distribution is close to Gaussian, especially if N is large enough. This also holds for α-stable distribution with α=1.5.

In the case of Gaussian PDF, σ_β and MAD_β for δ/MAD<0.1 are by approximately two times larger than δ/MAD>100. There is no minimums of the curves $\sigma_\beta(\delta)$ and $MAD_\beta(\delta)$. Both curves are monotonically decreasing. Both $\sigma_\beta(\delta)$ and $MAD_\beta(\delta)$ are proportional to σ_G and they are approximately inversely proportional to $\sqrt{N}$. The following condition holds: $\sigma_\beta(\delta) \approx 1.5MAD_\beta(\delta)$. This indicates that the distribution of estimates are close to Gaussian.

Let us analyze data obtained for Cauchy PDF, N=64. The dependences $\sigma_\beta(\delta)$ and $MAD_\beta(\delta)$ are presented in Fig. 2,a. As it is seen, these functions are monotonously decreasing. For δ/MAD<0.1, σ_β and MAD_β are by approximately two times larger than for δ/MAD_X>10 where $MAD_X = \mathrm{median}\{|\mathrm{x_i} - \mathrm{median}\{\mathrm{x_i}\}|\,,\ \mathrm{i} = 1,...,\mathrm{N}\}$ for a data sample at hand. Again $\sigma_\beta(\delta)$ and $MAD_\beta(\delta)$ are proportional to data scale defined by γ. For providing the best accuracy of the meridian estimator it is enough to set $\delta = (10...20)MAD_X$. Thus, a common conclusion for Gaussian and Cauchy PDFs is that accuracy of the meridian estimator cannot be better than for median estimator.

Let us now study statistical characteristics of meridian estimator for peaky PDFs starting from the PDF $f_2(x)$. The obtained dependencies are represented in Fig. 2,b. Both curves $\sigma_\beta(\delta)$ and $MAD_\beta(\delta)$ have minima observed for $\delta \approx 1$ and $\delta \approx 0.1$, respectively. Note also that the ratio $MAD_\beta/\sigma_\beta < 0.5$, i.e., considerably smaller than if meridian estimator is applied to Gaussian data. This indirectly shows that the estimates $\hat{\beta}_j$, $j = 1,...,N_{\exp}$ do not obey Gaussian distribution and are heavy tailed. This has been confirmed by analysis of their histograms.

For data samples with PDF $f_3(x)$, the plots $\sigma_\beta(\delta)$ and $MAD_\beta(\delta)$ are given in Fig. 3,a. There are obvious minimums of both σ_β and MAD_β observed for $\delta \approx 0.1$ and $\delta \approx 0.01$, respectively. For practice, it is possible to recommend using $\delta/MAD_{Y2} \approx 0.01$ that clearly corresponds to modal mode of the meridian estimator. Note that MAD_β/σ_β for $\delta/MAD_{Y2} \approx$ 0.01 becomes about 0.3. This means that PDF of meridian estimates is non-Gaussian.

The dependencies $\sigma_\beta(\delta)$ and $MAD_\beta(\delta)$ for the PDF $f_4(x)$ are represented in Fig. 3,b.

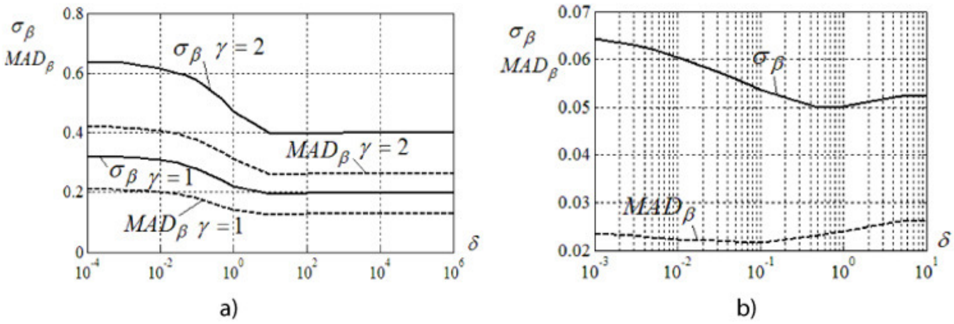

Fig. 2. Dependences $\sigma_\beta(\delta)$ and $MAD_\beta(\delta)$ for Cauchy PDF for $\gamma = 1$ and $\gamma = 2$ (a) and PDF $f_2(x)$ (b), N=64.

Minimum of $\sigma_\beta(\delta)$ has place for $\delta \approx 0.001$, minimum of $MAD_\beta(\delta)$ is observed for $\delta \approx 0.00001$, i.e. again the meridian estimator looks for distribution mode. Thus, δ/MAD_{Y3} should be about 0.0001. Note that σ_β for δ/MAD_{Y3}=1 is considerably larger than σ_β for the aforementioned optimum. This means that the meridian estimator with properly adjusted δ/MAD_{Y3} is able to perform considerably better than the median estimator. MAD_β/σ_β occurs to be very small, about 0.02. This means that meridian estimates have very heavy tails.

Interesting results have been obtained for the α-stable distributions with α=0.7 and α=1.5. The dependences $\sigma_\beta(\delta)$ and $MAD_\beta(\delta)$ for them are presented in Fig. 4. The curves for α=1.5 do not have minimums. At the same time, the curves for α=0.7 have minima observed for $\delta \approx 10$. This means that the sample meridian provides slightly better accuracy than the sample median for very heavy tail α-stable distributions under condition that the sample meridian tunable parameter is properly adjusted.

For many widely used distributions (Gaussian, Rayleigh, uniform) and conventional estimators it is commonly considered that an estimate RMSE is proportional to $1/\sqrt{N}$ [2, 10]. But for heavy tailed PDFs analyzed in our paper there is another tendency. Table 1 presents simulation results for PDFs $f_2(x)$, $f_3(x)$, and $f_4(x)$ for three values of N equal to 32, 64, and 256 (these values are typical for robust DFT applications [5]). The results are obtained for quasi-optimal δ_{qopt} that correspond to minimal σ_β and MAD_β. As it is seen, σ_β and MAD_β decrease faster than $1/\sqrt{N}$, especially for data samples with the most heavy tailed PDF $f_4(x)$. It is also worth stressing that optimal δ_{qopt} decreases if N becomes larger. This is a specific property that has not been observed for the myriad estimator [13]. This property makes more complicated designing an adaptive algorithm for determination of δ_{qopt} for limited a priori information on PDF a data sample obeys to.

We have also analyzed dependence of δ_{qopt} on data scale. As expected, experiments carried out for all considered PDFs have demonstrated that δ_{qopt} should be directly proportional to MAD of a given distribution where MAD_X can serve as its estimate.

It could be also interesting of what can be the benefit due to applying the meridian estimator with δ_{qopt} (given in Table 1) to heavy tail PDFs in comparison to the mean and median estimators. Their accuracy can be characterized by RMSE σ_{mean} and σ_{med}, respectively. The obtained data are given in Table 2. It can be also analyzed in terms of the corresponding MAD values MAD_{mean}, MAD_β, MAD_{med}. The results are presented in Table 3.

Analysis of simulation data presented in Tables 2 and 3 shows that the use of the meridian estimator with δ_{qopt} is expedient for all three considered heavy tailed peaky PDFs, especially for data samples that obey PDFs $f_3(x)$ and $f_4(x)$ and especially if a data sample size is large. For the latter two cases, σ_β can be smaller than σ_{med} by several times and even

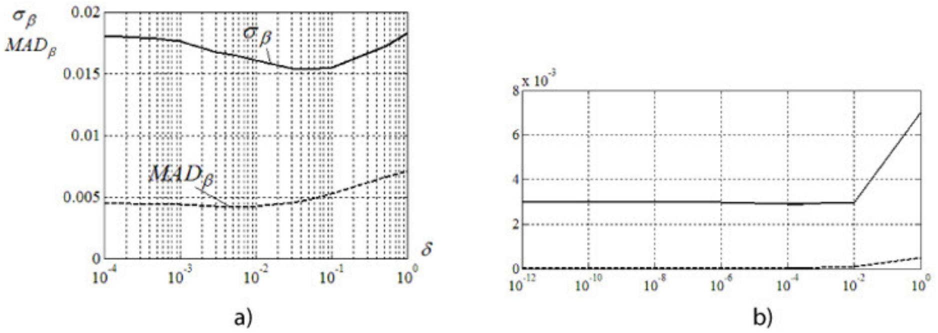

Fig. 3. Dependencies $\sigma_\beta(\delta)$ and $MAD_\beta(\delta)$ for the PDFs $f_3(x)$(a) and $f_4(x)$ (b), N=64, $\sigma_{X1} = \sigma_{X2} = \sigma_{X3} = 1$.

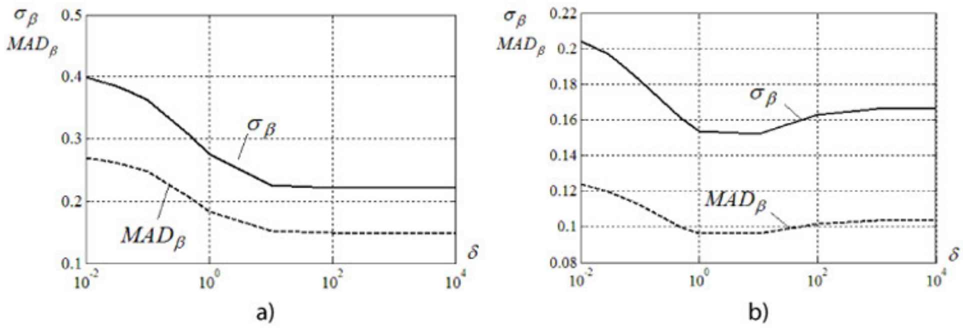

Fig. 4. Dependences $\sigma_\beta(\delta)$ and $MAD_\beta(\delta)$ for α-stable distributions with α=0.7 (a) and α=1.5 (b), $\gamma = 1$, N=64.

TABLE I

Accuracy of meridian estimator depending upon N.

PDF	$f_2(x)$			$f_3(x)$			$f_4(x)$		
N	δ_{qopt}	σ_β	MAD_β	δ_{qopt}	σ_β	MAD_β	δ_{qopt}	σ_β	MAD_β
32	0.5	0.084	0.038	0.05	0.032	0.0080	0.00005	0.012	0.00017
64	0.3	0.050	0.023	0.03	0.016	0.0044	0.00003	0.0015	0.000034
256	0.03	0.018	0.0074	0.001	0.0036	0.0011	10^{-8}	3.3õ10^{-5}	3.8õ10^{-7}

by tens of times. The same relates to MAD_β and MAD_{med}. Note that for the PDFs $f_2(x)$, $f_3(x)$, and $f_4(x)$ the median estimator provides better accuracy than the myriad estimator with optimal k. Thus, the meridian estimator can be useful for applications where one deals with very impulsive noise environments for which noise PDF is not bell-shaped. We also expect that the properties of the meridian and median estimators can differ a lot in cases of processing data samples with asymmetric distributions.

IV. Conclusions

The studies carried out have shown that there exist non-Gaussian distributions for which the sample meridian with δ_{qopt} is able to produce more accurate estimates of LP than the most known robust estimators, sample median and myriad. All three PDFs for which obvious benefits of the sample meridian have been observed are peaky (not bell-shaped) ones. This is explained by the peculiarities of the used cost function (2). For PDFs with heavier tails one has to set smaller δ_{qopt}. At the same time, δ_{qopt} should be pro-

TABLE II

Comparison of accuracy for the mean, median and meridian estimators for data samples with different N and PDFs

PDF	$f_2(x)$			$f_3(x)$			$f_4(x)$		
N	σ_{mean}	σ_{β}	σ_{med}	σ_{mean}	σ_{β}	σ_{med}	σ_{mean}	σ_{β}	σ_{med}
32	0.177	0.084	0.083	0.173	0.032	0.042	0.71	0.012	0.045
64	0.130	0.050	0.055	0.122	0.016	0.023	0.46	0.0015	0.015
256	0.062	0.018	0.021	0.063	0.0036	0.0070	0.24	3.3õ10^{-5}	0.0018

TABLE III

Comparison of accuracy for the mean, median and meridian estimators for data samples with different N and PDFs

PDF	$f_2(x)$			$f_3(x)$		
N	MAD_{mean}	MAD_{β}	MAD_{med}	MAD_{mean}	MAD_{β}	MAD_{med}
32	0.120	0.037	0.046	0.108	0.008	0.015
64	0.084	0.021	0.028	0.082	0.0043	0.0084
256	0.041	0.007	0.012	0.040	0.00096	0.0034

PDF	$f_4(x)$		
N	MAD_{mean}	MAD_{β}	MAD_{med}
32	0.44	2.0õ10^{-4}	0.0047
64	0.31	2.5õ10^{-5}	0.0015
256	0.15	1.1õ10^{-6}	0.00013

portional to data scale. These observations let us hope that an adaptive algorithm for determination δ of the meridian estimator can be designed.

References

[1] Spaulding, A.D., Middleton, D. Optimum reception in an impulsive interference environment. Part I: Coherent Detection. IEEE Trans. Commun. 25, (9), 910-923, 1977.

[2] Huber P., Robust statistics, *Wiley*, Stateplace-New York, 1981.

[3] Astola J., Kuosmanen P. Fundamentals of nonlinear digital filtering, Boca Raton (USA), CRC Press LLC, 1997, 276.

[4] Kalluri S., Arce G. Adaptive Weighted Myriad Filter Algorithms for Robust Signal Processing in α-stable Noise Environments, IEEE Trans. on Signal Processing, 1998, Vol.46, No 2, pp. 322-334.

[5] Djurovic I., Lukin V., Robust DFT with high breakdown point for complex valued impulse noise environment, IEEE Signal Processing Letters, Vol. 13, No 1, 2006, pp. 25-28.

[6] V.V. Lukin, S.K. Abramov, A.A. Zelensky, J.T. Astola, Myriad based shift parameter estimation method and its application to image filtering and processing, Proceedings of SPIE Conference "Mathematical Methods in Pattern and Image Analysis", San Diego (USA), July-Aug. 2005, SPIE Vol. 5916, pp. 1-12.

[7] Taguchi A., Adaptive α-Trimmed Mean Filter with Excellent-Detail Preserving, Proceedings of ICASSP, 1994, pp. 61-64.

[8] Lim Heng Siong, Chuah Teong Chee, Chuah Hean Teik, On the Optimal Alpha-k Curve of the Sample Myriad, IEEE Signal Processing Letters, 2007, Vol. 14, No 8, pp. 545-548.

[9] T.C. Aysal, K.E. Barner, CityplaceMeridian Filtering for Robust Signal Processing, IEEE Transactions on Signal Processing, Vol. 55, No 8, 2007, pp. 3949-3962.

[10] Two-Dimensional Digital Signal Processing II. Transforms and Median Filters. Edited by T.S. Huang, Springer-Verlag, 1981.

[11] V. Lukin, A. Totsky, D. Fevralev, A. Roenko, J. Astola, K. Egiazarian, Adaptive Combined Bispectrum-Filtering Signal Processing in Radar Systems with Low SNR, Proceedings of ISCAS, Kos Island, Greece, 2006, pp. 3690-3693.

[12] Katkovnik V., Robust M-estimates of the frequency and amplitude of a complex-valued harmonic, Signal Processing, Vol. 77, 1999, pp. 71-84.

[13] Roenko A., Lukin V., Djurovic I., Abramov S., Adaptation of Sample Myriad Tunable Parameter to Characteristics of SaS Distribution, Proceedings of the Conference on Mathematical Methods in Electromagnetic Theory (MMET 2008), June 2008, Odessa, Ukraine, pp. 418-420.

[14] Suoranta R. Amplitude domain approach to digital filtering. Theory and applications: Thesis for the degree of Doctor of Technology, Tampere (Finland, Tampere University of Technology), 1995, 199 p.

[15] Aysal T.C., Barner K.E. Second-Order Heavy-Tailed Distributions and Tail Analysis, IEEE Trans. on Signal Processing, Vol. 54, No. 7, 2006, pp. 2827-2832.