Modification of the robust chirp-rate estimator for impulse noise environments

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Abstract— **A modification of the robust chirp-rate estimator is proposed. The proposed technique has large breakdown point resulting in robustness to high amount of the impulse noise. In this approach, robust filtering of the polynomial-phase signals (PPS) is performed in an initial stage followed by the standard chirp-rate estimation for filtered signal. Numerical examples confirm accuracy of the proposed technique with pre-filtering in the initial stage.**

I. Introduction

The parameter estimation of polynomial phase signals (PPS) is an important issue in numerous research areas. The main research stream are so-called phase differencing techniques considered by numerous authors [1]-[6]. There are several alternatives including novel differential-equation based approach from [7]. Recently, O'Shea has proposed an innovative technique for chirp-rate estimation that includes non-linearity of the second-order in the transform [8]. Several improvements and generalization of this transform are proposed in [8]-[14]. This estimator is developed for the Gaussian noise environment and it does not produce accurate results for the impulse noise environments. We have proposed a robust form of the O'Shea estimator that can be used for impulse and mixed Gaussian and impulse environments [12]. In the same direction, we develop a modification that is able to produce accurate results for impulse noise environment with higher percentage of impulses than in the case of the initially proposed technique, i.e., the proposed chirp-rate estimator has higher breakdown point than the previous technique.

The manuscript is organized as follows. In

Section II, the chirp-rate estimator and the robust chirp-rate estimator are reviewed. The proposed modification is introduced in Section III. Numerical examples are given in Section IV.

II. Chirp-rate estimator

A. Basic form

Consider a PPS $x(t) = A\exp(j\phi(t))$. The first derivative of the signal phase is defined as the instantaneous frequency (IF) $\omega(t) = \phi'(t)$. A class of the IF estimators is based on the time-frequency (TF) analysis [15]-[17]. Consider, for example, the Wigner distribution (WD) here given in windowed (pseudo) discrete-time form:

$$WD(t,\omega) = \sum_{n=-\infty}^{\infty} w(nT)$$

$$\times x(t+nT)x^*(t-nT)e^{(-j2\omega nT)}, \quad (1)$$

where T is the sampling interval and $w(nT)$ is the window function. The IF can be estimated by using peaks of the WD as:

$$\hat{\omega}(t) = \arg\max_{\omega} WD(t,\omega). \quad (2)$$

Note that the phase of the local autocorrelation $x(t+nT)x^*(t-nT)$ using the modified Taylor's series expansion can be written as:

$$\Phi(t,nT) = \phi(t+nT) - \phi(t-nT) \approx$$

$$\approx 2\phi'(t)(nT) + \phi'''(t)\frac{(nT)^3}{3} + \phi^{(5)}(t)\frac{(nT)^5}{60} + ...,$$
$$(3)$$

where $\phi^{(a)}(t)$ denotes the a-th derivative of the signal phase. If higher-order phase terms are

equal to 0 i.e., $\phi^{(a)}(t) = 0$ for $a > 2$, the WD is ideally concentrated on the IF. As a result, a well-known central phase difference formula can be approximated as:

$$\phi'(t) \approx \frac{\phi(t + nT) - \phi(t - nT)}{2(nT)}. \quad (4)$$

Estimation of the higher-order phase terms is also very important [1]-[6], [18], [19]. In general, it is required to have higher order non-linearity in the estimator. However, the non-linearity causes degradation of the estimate performance with respect to additive noise influence.

Similarly, the difference equation for estimation of the chirp-rate parameter (the second-derivative of the phase) can be written as:

$$\phi''(t) \approx \frac{\phi(t + nT) - 2\phi(t) + \phi(t - nT)}{(nT)^2}, \quad (5)$$

which corresponds to the local auto-correlation function $x(t + nT)x^{*2}(t)x(t - nT)$. It is noted that $x^{*2}(t)$ does not depend on nT neither does the magnitude of the local auto-correlation function. Therefore, the chirp-rate can be estimated by using the nonlinear kernel $x(t + nT)x(t - nT)$ as [8]:

$$C(t, \Omega) = \sum_{n=-\infty}^{\infty} w(nT)$$

$$\times x(t + nT)x(t - nT)e^{(-j\Omega(nT)^2)}, \quad (6)$$

where $C(t, \Omega)$ is referred as the cubic phase function (CPF), and Ω denotes chirp-rate index. Estimation of the chirp-rate can then be performed as:

$$\hat{\Omega}(t) = \arg\max_{\Omega} |C(t, \Omega)|. \quad (7)$$

In this manner non-linearity for the chirp-rate estimation is kept the same as in the case of the WD, i.e., the second-order. It results in high accuracy approaching toward the Cramer-Rao lower bound (CRLB) for a wide range of the signal-to-noise ratios (SNR) for Gaussian noise environment [8], [9], [11]. However, this technique in the presented form cannot be used for impulse noise environments.

B. Robust form

The CPF (6) can be written in the alternative form:

$$C(t, \Omega) = \text{mean}\{x(t + nT)x(t - nT)$$

$$\times e^{(-j\Omega(nT)^2)} \mid n \in [-N/2, N/2]\}. \quad (8)$$

Here, we assume that the window function is $w(nT) = 1/(N + 1)$ for $n \in [-N/2, N/2]$ and $w(nT) = 0$ elsewhere. This form suffers from the same troubles as the linear moving average filter in signal filtering. Namely, all samples of modulated auto-correlation $x(t + nT)x(t - nT)\exp(-j\Omega(nT)^2)$ are taken with the same weights. Samples corrupted with impulse noise can significantly disturb this function and at the same time they can worsen accuracy of the CPF. Therefore, the robust form of the CPF is proposed in [12] where the L-filter form of the chirp-rate estimator is introduced:

$$C_L(t, \Omega) = \sum_{l=-N/2}^{N/2} a_l[\mathbf{r}_{(l)}(\mathbf{t}, \mathbf{\Omega}) + \mathbf{ji}_{(l)}(\mathbf{t}, \mathbf{\Omega})] \quad (9)$$

where $\mathbf{r}_{(l)}(\mathbf{t}, \mathbf{\Omega}) \in \mathbf{R}(\mathbf{t}, \mathbf{\Omega})$ and $\mathbf{i}_{(l)}(\mathbf{t}, \mathbf{\Omega}) \in \mathbf{I}(\mathbf{t}, \mathbf{\Omega})$. Sets $\mathbf{R}(\mathbf{t}, \mathbf{\Omega})$ and $\mathbf{I}(\mathbf{t}, \mathbf{\Omega})$ are formed as:

$$\mathbf{R}(\mathbf{t}, \mathbf{\Omega}) = \{\text{Re}\{\mathbf{x}(\mathbf{t} + \mathbf{nT})\mathbf{x}(\mathbf{t} - \mathbf{nT})$$

$$\times e^{(-j\Omega(nT)^2)}\} \mid n \in [-N/2, N/2]\},$$

$$\mathbf{I}(\mathbf{t}, \mathbf{\Omega}) = \{\text{Im}\{\mathbf{x}(\mathbf{t} + \mathbf{nT})\mathbf{x}(\mathbf{t} - \mathbf{nT})$$

$$\times e^{(-j\Omega(nT)^2)}\} \mid n \in [-N/2, N/2]\}, \quad (10)$$

Elements $\mathbf{r}_{(l)}(\mathbf{t}, \mathbf{\Omega})$ and $\mathbf{i}_{(l)}(\mathbf{t}, \mathbf{\Omega})$ from the corresponding sets are sorted into a non-decreasing order:

$$\mathbf{r}_{(l)}(\mathbf{t}, \mathbf{\Omega}) \leq \mathbf{r}_{(l+1)}(\mathbf{t}, \mathbf{\Omega}), \quad (11)$$
$$\mathbf{i}_{(l)}(\mathbf{t}, \mathbf{\Omega}) \leq \mathbf{i}_{(l+1)}(\mathbf{t}, \mathbf{\Omega}).$$

Parameters of the L-filter are selected as: $\sum_{l=-N/2}^{N/2} a_l = 1$ (energy condition) and $a_l = a_{-l}$ (unbiasedness condition). We select these parameters according to the α-trimming rule:

$$a_l = \begin{cases} 1/(2Na + 1) & l \in [-aN, aN] \\ 0 & \text{elsewhere,} \end{cases} \quad (12)$$

where for $a = 1/2$ we obtain the standard chirp-rate estimator (8) while for $a \in [0, 1/2)$ we obtain the robust form where some percentage of the samples with the highest values is removed.

III. PROPOSED MODIFICATION

Consider the following signal model:

$$x(t) = f(t) + \nu(t),$$

where $f(t) = A \exp(j\phi(t))$, while the noise can be written as: $\nu(t) = \nu_1(t) + j\nu_2(t)$ where impulses appear in both $\nu_1(t)$ and $\nu_2(t)$ with probability p ($p/2$ is probability of both positive and negative impulses). Impulses have the same magnitude. We further assume that $\nu_i(t)$, $i = 1, 2$ are mutually independent i.i.d. processes, i.e., $E\{\nu_i(t')\nu_j(t'')\} = p\beta^2\delta(t' - t'')\delta(i - j)$, $i = 1, 2$, where β is amplitude of pulses. The real part of $\text{Re}\{x(t + nT)x(t - nT)\exp(-j\Omega(nT)^2)\}$ can be calculated as:

$$\text{Re}\{x(t + nT)x(t - nT)\exp(-j\Omega(nT)^2)\} =$$

$$= \text{Re}\{x(t + nT)x(t - nT)\}\cos(\Omega(nT)^2)$$

$$+ \text{Im}\{x(t + nT)x(t - nT)\}\sin(\Omega(nT)^2) =$$

$$= [r(t + nT)r(t - nT) - i(t + nT)i(t - nT)]$$

$$\times \cos(\Omega(nT)^2)$$

$$+ [r(t + nT)i(t - nT) + i(t + nT)r(t - nT)]$$

$$\times \cos(\Omega(nT)^2). \quad (13)$$

For $n \neq 0$ in the resulting sequence probability of appearance of pulses is approximately $4p$ (for small p where $p^2 \ll p$). This fact results in significant drawbacks in the robust estimate (9). Therefore, high percentage of pulses degrades performance of the robust transforms and, in addition, it reduces a so-called breakdown point of the algorithm [20]. On one hand a small trimming parameter a is preferred in order to avoid impulses for high percentage of pulses. On the other hand, a small value of a causes the spectral distortion effect in the estimate of the CPF [21] and, consequently, it reduces estimator accuracy [22]. Specifically, for relatively modest percentage of pulsed $p \geq 1/8$, where the resulting signal

of (13) has probability of impulse noise appearance greater than $1/2$, robust estimate (9) is unable to produce accurate results.

In this paper, the goal is to develop a method that would reduce the influence of impulses to the CPF. For this purpose a two-step procedure is proposed. Firstly, filtering of pulses is performed by using the robust DFT based filter [21],[22]. In the second step, the standard CPF evaluation of the filtered signal is performed. This procedure can be described as:

$$x(t) \to \hat{x}(t)$$

$$= IDFT\{\hat{X}(\omega)\} \to CPF_{\hat{x}}(t, \Omega) \to \hat{\Omega}(t), \quad (14)$$

where $CPF_{\hat{x}}(t, \Omega)$ is the standard CPF (8) calculated for $\hat{x}(t)$, a signal obtained using the robust DFT-based filtering. In order to reduce the impulses as much as possible, the robust DFT form with high breakdown point proposed in [22] is applied. The robust DFT form can be calculated as:

$$\hat{X}(\omega) = \hat{X}_1(\omega) + \hat{X}_2(\omega) + j[\hat{X}_3(\omega) + \hat{X}_4(\omega)], \quad (15)$$

where

$$\hat{X}_i(\omega) = \sum_{l=-N/2}^{N/2} a_l \mathbf{r}_{(\mathbf{i},\mathbf{l})}(\boldsymbol{\omega}) \quad (16)$$

while $\mathbf{r}_{(\mathbf{i},\mathbf{l})}(\boldsymbol{\omega})$ are sorted elements from the sets:

$$\mathbf{r}_{(\mathbf{i},\mathbf{l})}(\boldsymbol{\omega}) \in \mathbf{R}_{\mathbf{i}}(\boldsymbol{\omega})$$

$$= \{r_i(nT, \omega)|n \in [-N/2, N/2]\},$$

$$i = 1, 2, 3, 4, \quad (17)$$

where $r_1(nT, \omega) = \text{Re}[x(nT)]\cos(\omega(nT))$, $r_2(nT, \omega) = \text{Im}[x(nT)]\sin(\omega(nT))$, $r_3(nT, \omega) = -\text{Re}[x(nT)]\sin(\omega(nT))$ and $r_4(nT, \omega) = \text{Im}[x(nT)]\cos(\omega(nT))$. In this way all four quantities $\hat{X}_i(\omega)$, $i = 1, ..., 4$, are calculated by using modulated sequences with the same percentage of pulses. This leads to the signal filtered by using transform $\hat{X}(\omega)$ with high breakdown point and with significantly reduced influence of impulse terms. Namely, percentage of pulses in transforms $\hat{X}_i(\omega)$, $i = 1, ..., 4$, is equal to p, i.e., it is significantly reduced with respect to (9). Then, the chirp-rate is estimated by using the CPF of filtered signal $\hat{x}(t)$.

IV. Numerical examples

We consider a signal

$$x(n) = Ae^{\left(ja_3n^3/6 + ja_2n^2/2 + ja_1n + ja_0\right)}. \quad (18)$$

Parameters of the signal are selected as $A = 1$, $a_0 = 1$, $a_1 = \pi/5$, $a_2 = \pi/(5N)$ and $a_3 = -\pi/(8N^2)$. Signal is considered in the interval $n \in [-(N-1)/2, (N-1)/2]$ where $N = 513$. The chirp-rate of this signal is $\Omega(n) = a_2 + a_3n$. Estimation is performed for middle of the interval $n = 0$ where the chirp-rate is $\Omega(0) = a_2$.

Four types of noise environments are considered in our study.

Example 1. The first noise environment is a mixture of the Gaussian noise and impulsive noise of the salt and pepper type:

$$\begin{aligned} \nu(n) &= \sigma(\nu_1(n) + j\nu_2(n))/\sqrt{2} + \quad (19) \\ &\quad + \beta(\xi_1(n) + j\xi_2(n)), \end{aligned}$$

where $\nu_i(n)$, $i = 1, 2$, are mutually non-correlated Gaussian noises with zero-mean and unitary variance, $E\{\nu_i(n)\} = 0$, $E\{\nu_i(n)\nu_j(n)\} = \delta(i-j)$, $i, j = 1, 2$. Impulsive noise components are denoted as $\xi_i(n)$, $i = 1, 2$ where $\xi_i(n) = 0$ and $E\{\xi_i(n)\xi_j(n)\} = 0$ for $i \neq j$. We assume that percentage of pulses is p (equal probability of both positive and negative pulses) where amplitude is set to unity. Strength of pulses is controlled with the parameter β. In our experiments we set $\beta = 5$.

Figure 1 depicts root mean squared estimation errors (MSE) for different robust DFT forms as a function of probability of impulse noise for fixed amount of the Gaussian noise. The results are given for SNR=$10\log_{10}(A^2/\sigma^2)$ equal to -6dB, -3dB, 0dB and 3dB respectively (hereafter we assume the SNR as the ratio between signal power and variance of Gaussian noise component). The accuracy is compared for the standard CPF, the robust CPF with $a = 1/4$, the median CPF form, and for the proposed technique (denoted with 'New' with given parameter of the robust DFT used for signal filtering). It can be seen that the improvement achieved with the proposed technique for large amount of the Gaussian noise (Fig. 1a) is quite moderate and it exhibits

about 1.1dB with respect to the standard robust CPF and about 0.9dB with respect to the robust DFT for $p = 0.2$. However, this improvement is quite large in the case of the smaller amount of the Gaussian noise component. For example, the improvement of the proposed technique for $p = 0.2$ and SNR=-3dB (see Fig. 1b) with respect to the standard CPF is about 12dB while with respect to the robust CPF from [12] it is about 3dB. Improvement is even more emphatic for smaller amount of the Gaussian noise.

Example 2. The second, more realistic, noise model is sum of Gaussian with cube of Gaussian noise:

$$\begin{aligned} \nu(n) &= \sigma(\nu_1(n) + j\nu_2(n))/\sqrt{2} + \\ &\quad \alpha(\nu_3^3(n) + j\nu_4^3(n))/\sqrt{2} \quad (20) \end{aligned}$$

where $\nu_i(n)$, $i = 1, 2, 3, 4$, are mutually non-correlated Gaussian noises with zero-mean and unit variance, $E\{\nu_i(n)\} = 0$, $i = 1, 2, 3, 4$, $E\{\nu_i(n)\nu_j(n)\} = \delta(i-j)$. Simulation results are given in Fig. 2 which shows the root MSE as a function of α for a fixed amount of Gaussian noise component. Obtained results show that the proposed technique has significantly improved accuracy with respect to the robust CPF from [12] and with respect to the standard CPF form.

Example 3. Recently, a very commonly used model of the heavy tailed noise is the symmetric α-stable noise [23]. This noise is characterized with two parameters (α, γ) where $\alpha \in [0, 2]$. Smaller α implies more impulsive noise. Parameter γ corresponds to the noise strength. The α-stable noise has been successively applied for numerous phenomena appearing in practical applications, for example: atmospheric noise, Internet traffick modeling, noise in video-sequences, etc. Results of the Monte-Carlo simulations for $\alpha = 0.4$, $\alpha = 0.8$, $\alpha = 1.2$, and $\alpha = 1.6$ as function of $\gamma \in [1, 10]$ are depicted in Fig. 3. Again it can be seen that the proposed modification produces significantly better results than the robust CPF from [12]. This improvement is more emphatic for heavier impulsive environment (smaller α).

Example 4. Several studies have shown the importance of designing impulsive noise gen-
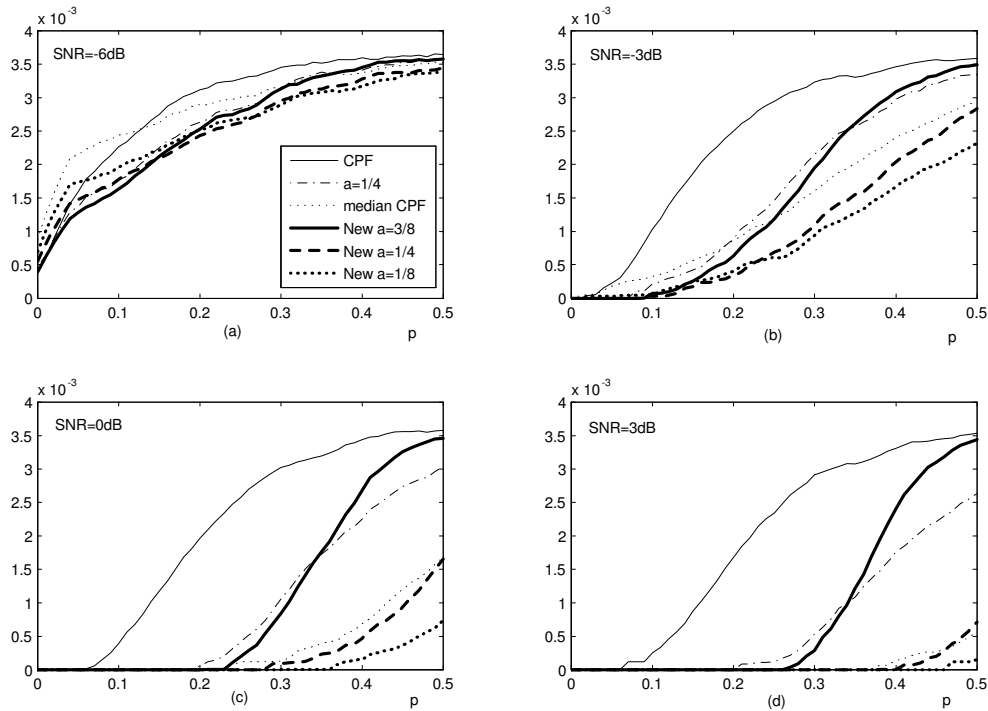
Fig. 1. Root-mean squared error in chirp rate estimation with different cubic phase function forms for mixed Gaussian and impulsive noise environment as function of percentage of impulsive noise for fixed amount of Gaussian noise component.

erator with characteristics accurately corresponding to the practical noise environments [24], [?]. In [25], an impulsive generator for telephone networks of the British Telecom and Deutsche Telekom with appropriate amplitude, length, inter-arrival and spectral analysis has been proposed. The Weibull noise with probability density function (here considered in the symmetric form) has been proposed as a proper model of the amplitude of impulses:

$$f(\zeta) = \frac{1}{2}\alpha\beta|\zeta|^{\alpha-1}e^{-\beta|\zeta|^{\alpha}}. \qquad (21)$$

In our experiment (Fig. 4a) the CPF forms are tested for the Weibull noise environment with fixed $\alpha = 1$ and varying β. It can be seen that the a proposed modification outperforms the other related CPF techniques.

In addition, it has been shown in [25], that the probability density function of the impulse width is a sum of two log-normal distributions. One of them corresponds to the impulses of relatively narrow width while the other corresponds to relatively wide impulses. In order to simulate similar environment we considered model from Example 1 with only impulsive noise component ($\sigma = 0$). The width of impulses is set to one sample with probability $0.8p$, two samples with probability $0.1p$, three samples with probability $0.05p$ and eight sample with probability $0.05p$. In this way we model noise environments with narrow and wide impulses. Obtained results as function of the total impulse noise probability p are depicted in Fig. 4b. Again, one can notice improvement of the proposed techniques with respect to previously proposed CPF forms.

V. CONCLUSION

We have demonstrated that improvement in the chirp-rate estimation can be achieved when the signal is pre-filtered with the robust DFT-based filters. The accuracy improvement can be observed with respect to both the standard
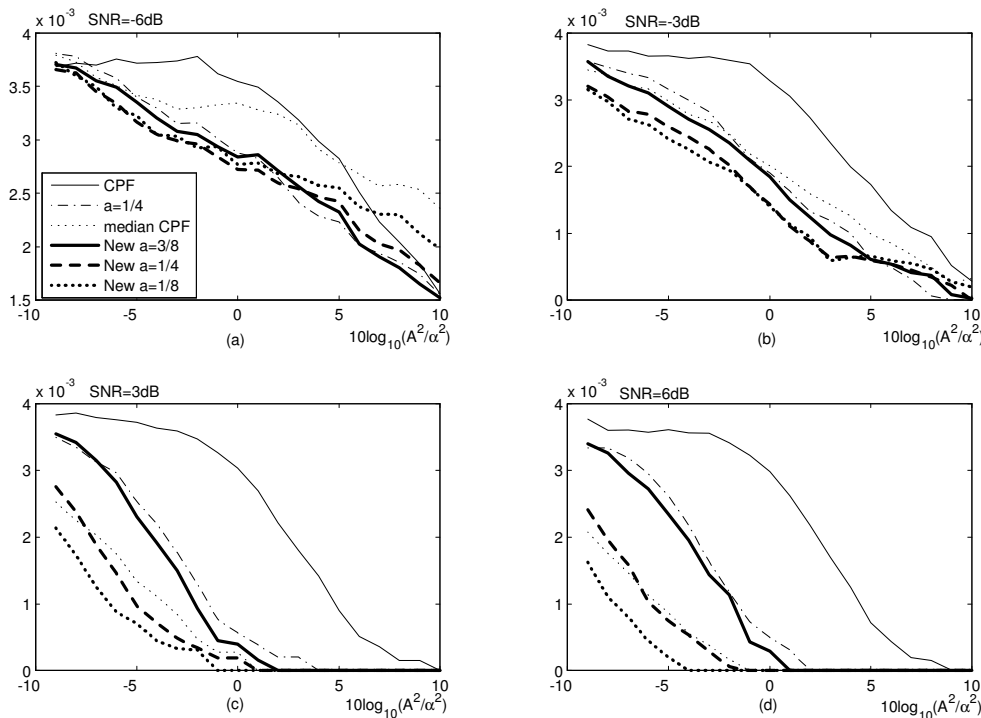
Fig. 2. Root-mean squared error in chirp rate estimation with different cubic phase function forms for mixed Gaussian and cube of Gaussian noise as function of amount of cube of Gaussian for fixed amount of Gaussian noise component.

CPF and the robust CPF where estimation function is calculated for auto-correlation of the signal. The proposed technique can be used in combination with other higher-order tools for parametric signal estimation in presence of impulsive noise.

VI. Acknowledgment

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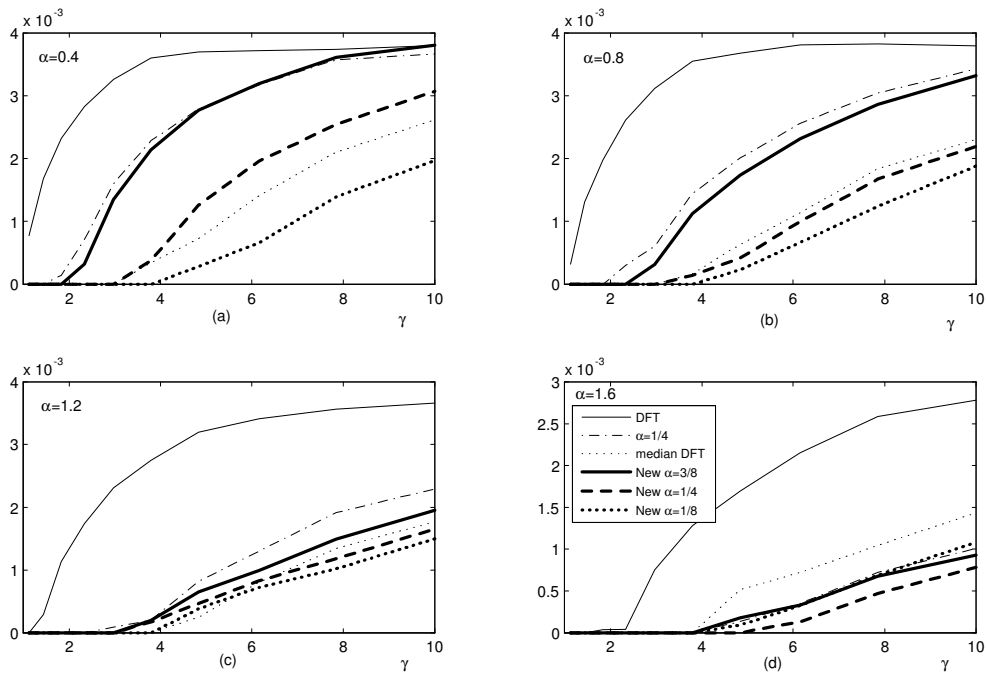
Fig. 3. Root-mean squared error in chirp rate estimation with different cubic phase function forms for $\alpha-$stable noise environment: (a) $\alpha = 0.4$; (b) $\alpha = 0.8$; (c) $\alpha = 1.2$; (d) $\alpha = 1.6$.
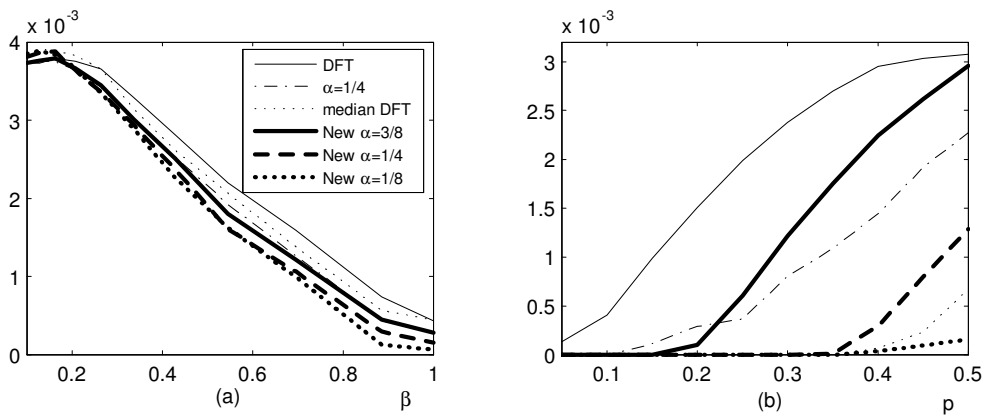

Fig. 4.   Root-mean squared error in chirp rate estimation with different cubic phase function forms for: (a) Weibull noise environment; (b) For noise environment with varying impulse noise widths.

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