A Virtual Instrument for Time–Frequency Analysis

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Abstract—A virtual instrument for time–frequency analysis is presented. Its realization is based on an order recursive approach to the time–frequency signal analysis. Starting from the short time Fourier transform and using the S-method, a distribution having the auto-terms concentrated as high as in the Wigner distribution, without cross-terms, may be obtained. The same relation is used in a recursive manner to produce higher order time–frequency representations without cross-terms. Thus, the introduction of this new virtual instrument for time–frequency analysis may be of help to the scientists and practitioners in signal analysis. Application of the instrument is demonstrated on several simulated and real data examples.

Index Terms— Instantaneous frequency and amplitude estimation, short-time Fourier transform, time–frequency analysis, virtual instrument, Winger distribution.

I. INTRODUCTION

TIME–FREQUENCY (TF) analysis is one of very important research and practical application areas in the signal analysis [1]. The oldest, simplest and most commonly used tool for TF analysis is the spectrogram (SPEC) via the short time Fourier transform (STFT). Implementations (hardware or software) of this transform are already widely present in practice [2]–[11]. Although very simple and convenient for applications, the STFT has some serious drawbacks. The most important one is its low concentration in the TF plane due to the contradictory conditions for high resolution in the time and frequency direction. In order to improve signal's TF representation, various quadratic distributions have been introduced [12]–[20]. The most important member of this class is the Wigner distribution (WD). However, the WD itself has a serious drawback. Namely, in the case of multicomponent signals it produces very emphatic cross-terms that can completely mask the auto-terms and make this distribution useless for analysis [7], [21]–[23]. This is why many other quadratic distributions have been introduced (Choi–Williams, Zao–Atlas–Marks, Born–Jordan, Butterworth, Zhang–Sato, etc.). Common goal to all of these distributions is to reduce cross-terms and other interferences, at the same time satisfying as many desired properties as possible. However, the numerical realization of the reduced interference distributions is more complex than the realization of the WD [10], [13], [24], [26], [27]. Also, the cross-term reduction inherently leads to the auto-terms degradation [17]. In order to preserve the auto-terms quality as in the WD, while reducing the cross-terms, the S-method (SM) has been introduced. By the SM these goals may be achieved in a numerically very efficient way. The SM realization is based on a direct application of the STFT. By adding one more stage to the already existing software or hardware systems for the STFT realization one can significantly improve the properties of TF representation.

Recently, virtual instrumentation started to play an important role in the signal analysis and signal measurements, including various instruments for TF analysis [28], [29]. "The trend today is for computers to serve as the engine for instrumentation. Virtual instruments leverage off the open architecture of industry standard computers to provide the processing, memory and display capacities; . . ." [28].

Here we propose a virtual instrument (VI) for TF analysis based on the SM realization. The VI contains the most important TF distributions: the SPEC and the WD, as special cases. In many applications it can combine good properties of both of these distributions, while avoiding their drawbacks. The presented VI can achieve concentration as high as in the WD, but without cross-terms between nonoverlaping components in the TF plane. According to the above facts we have found that the introduction of this new, simple, qualitatively and numerically very efficient instrument may be of help to the scientists and practitioners in signal analysis.

In the first part of the paper a review of the STFT and the WD is given. The SM, as a theoretical basis for the VI, is presented next. Algorithm for the VI, with its description, performance analysis, and examples, is presented in the last part of the paper.

II. REVIEW OF THE BASIC TF REPRESENTATIONS

A. Short-Time Fourier Transform

The STFT is defined by

$$\text{STFT}(t, \omega) = \int_{-\infty}^{\infty} x(t + \tau) w(\tau) e^{-j\omega\tau} d\tau \tag{1}$$

where $w(\tau)$ is a window function. Sliding this window along the signal $x(t)$ we get its frequency contents within a considered time interval determined by the window width. Obviously, the window form (especially its width) plays a crucial role in the signal analysis using the STFT. As it is known, the product of the window width in time domain and the width of its Fourier transform (or its main lobe) is constant for a given window function. For example, for $x(t) = \exp(j\omega_0 t) + \delta(t)$ we get $\text{STFT}(t, \omega) = \exp(j\omega_0 t)W(\omega - \omega_0) + \exp(j\omega t)w(-t)$. Improving resolution in one direction (t or ω) we ultimately worsen the resolution in the other direction (ω or t). Thus, the width of $w(\tau)$ has to be chosen by

an appropriate compromise. The SPEC, as an energetic version of the STFT, does not satisfy the marginal properties that state that the integral over frequency of an energetic distribution $P(t, \omega)$ should be equal to $|x(t)|^2$ (signal power), and the integral over time to $|X(\omega)|^2$ (spectral density), [6], [15]. In order to overcome these drawbacks, researchers look for more appropriate tools for TF analysis. One of them is the WD.

B. Wigner Distribution

Wigner distribution in its pseudo form is defined by

$$\text{WD}(t, \omega) = \int_{-\infty}^{\infty} w_e(\tau) x\left(t + \frac{\tau}{2}\right) x^*\left(t - \frac{\tau}{2}\right) e^{-j\omega\tau} d\tau \quad (2)$$

where the equivalent window is usually written as $w_e(\tau) = w(\tau/2)w(-\tau/2)$. For the previous example we get $\text{WD}(t, \omega) = w^2(0)\delta(t) + W_e(\omega - \omega_0)+$ cross-terms. Time resolution, in this case, is not dependent on the window form, while the frequency resolution may be arbitrary high if we use a *wide* window $w_e(\tau)$. This way, one may achieve arbitrary high resolutions in both directions, for the considered signal. The WD is the only one from the Cohen class that may produce a completely concentrated auto-term in the case of a linear frequency modulated (FM) signal $x(t) = A \exp(jat^2/2)$, when $\text{WD}(t, \omega) = A^2 W_e(\omega - at)$ [15], [17], [22]. These two examples demonstrate the WD superiority over the STFT. The WD satisfies many additional desired properties, including the marginal ones [6].

An annoying trait of the WD is the presence of cross terms when the multicomponent signals are analyzed [7], [15], [21], [22], [30]. A multicomponent signal is introduced as

$$x(t) = \sum_{m=1}^{M} x_m(t) = \sum_{m=1}^{M} A_m(t)e^{j\phi_m(t)}. \quad (3)$$

The STFT of $x(t)$ is equal to the sum of the STFT's of individual components. This very appealing property of the STFT is lost in the quadratic and higher order distributions. For (3) the WD has the form

$$\text{WD}(t, \omega) = \sum_{m=1}^{M} \int_{-\infty}^{\infty} w_e(\tau) x_m\left(t + \frac{\tau}{2}\right)$$
$$\cdot x_m^*\left(t - \frac{\tau}{2}\right)e^{-j\omega\tau}d\tau + \sum_{\substack{m=1 n=1 \\ m \neq n}}^{M} \sum^{m} \int_{-\infty}^{\infty}$$
$$\cdot w_e(\tau) x_m\left(t + \frac{\tau}{2}\right) x_n^*\left(t - \frac{\tau}{2}\right)e^{-j\omega\tau}d\tau. \quad (4)$$

Besides the auto-terms, the WD contains a significant number of cross-terms, for $m \neq n$. They appear due to the quadratic nature of the WD. Sometimes they can be so emphatic to completely cover the auto-terms.

III. VIRTUAL INSTRUMENT

A. Theoretical Background for the Instrument

Realization of the VI will be based on the distribution that can, in the case of multicomponent signals, produce a sum
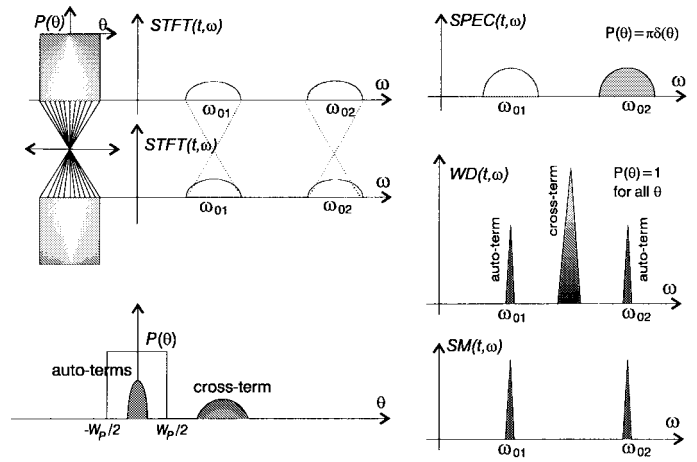

Fig. 1. Illustration of the SM calculation including two special cases: WD and spectrogram.

of the WD's of individual signal components, avoiding cross-terms. It will be referred to as the S-method, and defined as [10]

$$\text{SM}(t, \omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} P(\theta)\text{STFT}(t, \omega + \theta)$$
$$\cdot \text{STFT}^*(t, \omega - \theta) d\theta. \quad (5)$$

The SM produces the pseudo WD and the SPEC with $P(\theta) = 1$ and $P(\theta) = \pi\delta(\theta)$, respectively, as its special cases. The width of window $P(\theta)$ denoted by W_P ($P(\theta) = 0$ for $|\theta| > W_P/2$) should be wide enough to enable complete integration over the auto terms but, in order to avoid the cross terms, narrower than the distance between two auto-terms (Fig. 1). Then, according to (4) we get

$$\text{SM}(t, \omega) = \sum_{m=1}^{M} \int_{-\infty}^{\infty} w_e(\tau) x_m\left(t + \frac{\tau}{2}\right)$$
$$\cdot x_m^*\left(t - \frac{\tau}{2}\right)e^{-j\omega\tau} d\tau.$$

Window $P(\theta)$ may be also defined as signal dependent, when the only condition to get cross-terms free distribution is that the components do not overlap in the TF plane. One such realization is presented in [31]. Further details about the SM, including applications to the two-dimensional signals and affine distributions, may be found in [10], [17], [22], [23], [25], [31], [33].

The discrete form of the SM is given by [10], [27], [31]

$$\text{SM}(n, k) = \frac{2}{T_w} \sum_{i=-L_d}^{L_d} P(i)\text{STFT}(n, k + i)$$
$$\cdot \text{STFT}^*(n, k - i)$$
$$\text{STFT}(n, k) = \sum_{i=-N_w/2+1}^{N_w/2} x(n + i)w(i)e^{-j2\pi ik/N_w}T.$$
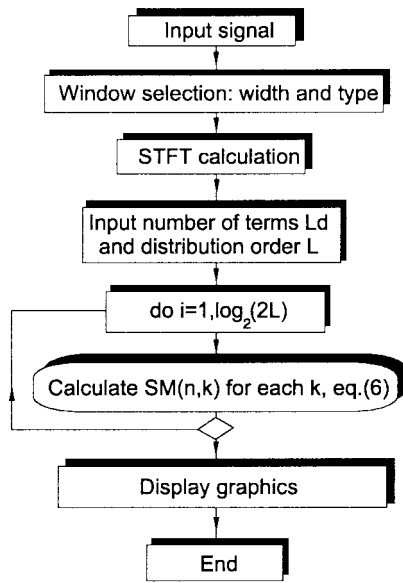
Fig. 2.   Simplified algorithm for the virtual instrument realization.

For the rectangular window $P(i)$ we get

$$\mathrm{SM}(n, k) = \frac{2}{T_w} \left[|\mathrm{STFT}(n, k)|^2 \right.$$
$$\left. + 2\mathrm{Re} \left\{ \sum_{i=1}^{L_d} \mathrm{STFT}(n, k+i) \mathrm{STFT}^*(n, k-i) \right\} \right] \quad (6)$$

where T is the sampling interval, and $T_w = N_w T$ is the width of $w(\tau)$. The oversampling is not necessary when the SM is used because the aliasing components are removed in the same way as are the cross-terms. Also, there is no need for analytic signal calculation since the cross-terms between negative and positive frequency components are removed in the same way as are the other cross-terms [22]. For the SM realization we have to implement the STFT first. This should be done in one of the well known ways, based either on the FFT routines or recursive approaches. Window $w(i)$ form, as well as its length, should be selected according to the widely known analysis of the STFT. After we get the STFT (and SPEC) we have to "correct" the obtained values, according to (6), by adding terms $2\mathrm{Re}\{P(i)\mathrm{STFT}(n, k+i)\mathrm{STFT}^*(n, k-i)\}$ to the SPEC values, Fig. 2. Theoretically, if we want to get the auto-terms with the same shape as in the WD, the number of added terms (L_d) should be large enough to include all additions inside an auto-term. However, we may significantly improve the TF representation with respect to SPEC (without knowing the auto-terms width) if we take only a few of these terms. A large number of terms usually does not produce any further improvement of the auto-terms concentration; it may only cause the occurrence of cross-terms as well as unnecessary accumulation of the noise [25].

In order to improve distribution concentration in the case of nonlinear FM signals, as well as to achieve some other

important properties, the higher order time-varying spectra have been defined [23]. The most interesting for practical realizations are the versions of these distributions that can be reduced to the TF plane. Here, we will present the L-WD [22], [23]

$$\mathrm{LWD}_L(t, \omega) = \int_{-\infty}^{\infty} x^{*L}\left(t - \frac{\tau}{2L}\right) x^L\left(t + \frac{\tau}{2L}\right) w_L(\tau)$$
$$\cdot e^{-j\omega\tau} \, d\tau.$$

For $L = 1$ it reduces to the WD. The realization of the cross-terms free L-WD may be efficiently done in the discrete domain using the recursive SM formula (6), Fig. 2,

$$\mathrm{LWD}_{2L}(n, k) = \frac{2}{T_w} \left[\mathrm{LWD}_L^2(n, k) + 2 \sum_{i=1}^{L_d} \right.$$
$$\left. \cdot \mathrm{LWD}_L(n, k+i) \mathrm{LWD}_L(n, k-i) \right]. \quad (7)$$

This is a very convenient form since the same blocks, connected in cascade, may provide a simple and efficient system. Modifications for the realization of the polynomial Wigner–Ville distributions are straightforward [32].

B. Instrument Implementation and Outlook

Based on the above analysis, we have realized a VI using the STFT and relations (6), (7). A simplified algorithm for this instrument realization is shown in Fig. 2. Lag window $w(i)$ parameters (its length and shape) should be determined according to the analysis of the STFT. After we select these parameters, the improvement of the distribution concentration may be achieved by increasing the number of terms (L_d) in (6). By increasing the distribution order L we may additionally improve the distribution concentration in the case of nonlinear FM signals.

The instrument panel is presented in Fig. 3. On the left hand side, along with the signal in time domain, we have its Fourier transform and TF distribution. On the right hand side there are six blocks that enable one to vary the most common presentation parameters, or to control the operations.

In the first block the lag window type and width are defined. Signal form and parameters are defined by the second block. Signal may be taken as: 1) a function of time t, and 2) data from a file (this option is convenient for real data analysis). Next block defines the number of terms in (6). The distribution order can be increased by using the fourth block.

Other blocks, together with the common MS Windows menu options, provide some additional possibilities for data, graphics and program control and manipulation. Instrument is implemented in MATLAB 5.

C. Performance Analysis

The performance analysis will be done by considering amplitude and instantaneous frequency (IF) estimation in the cases of both monocomponent and multicomponent signals.
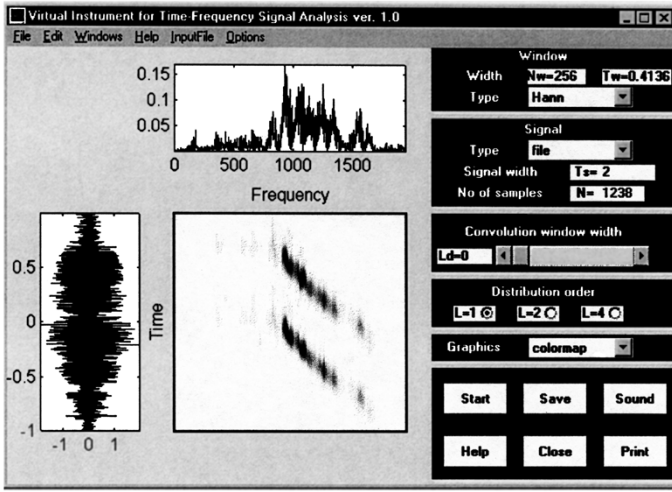
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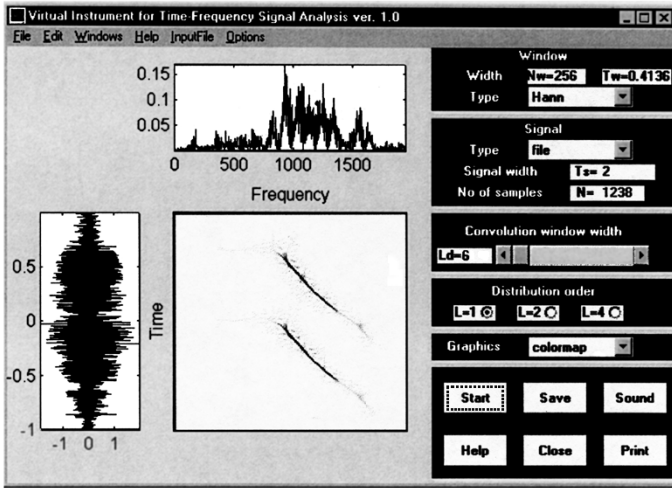
(b)

Fig. 3.   Time–frequency representation of a sonar signal: (a) spectrogram and (b) S-method. Relevant parameters are shown in the figure.

The estimation of amplitude and IF using TF representations have the following sources of errors:

1) input noise, including A/D quantization noise;
2) bias of a representation due to the input noise and time-variations;
3) leakage effects (on the amplitude) and quantization effects (on the IF) due to the discrete frequency grid;
4) mutual components influence in the case of the WD and multicomponent signals.

The *instantaneous frequency* is estimated as

$$k_m(n) = \arg \left\{ \max_k \{ WD(n, k) \} \right\}$$
$$\hat{\omega}(n) = k_m(n) \Delta\omega = \pi k_m(n)/T_w. \qquad (8)$$

The variance of the estimated IF $\hat{\omega}(n)$ is given by, [34]

$$\sigma_\omega^2 = \frac{\sigma_\nu^2}{4A^4(n)} (2A^2(n) + \sigma_\nu^2) G_w/(N_w T_w^2) \qquad (9)$$

where $G_w = \int_{-1/2}^{1/2} w_e^2(t)t^2 dt/(\int_{-1/2}^{1/2} w_e(t)t^2 dt)^2$ is a constant depending on the window form, while the variations of $A^2(n)$ are small within $w_e(n)$. For the Hanning window $w_e(n)$ we get $G_w = 28.1135$. The variance of input white noise is denoted by σ_ν^2.

Discrete frequency grid produces the quantization error with variance $\sigma_q^2 = (\Delta\omega)^2/12 = \pi^2/(12T_w^2)$. It may be reduced by additional interpolation along the frequency axis, or by using methods such as the one presented in [29]. The IF is then calculated as $\hat{\omega}(n) = \pi(k_m(n) + \delta(n))/T_w$, where the spectral displacement bin $\delta(n)$ for the Hanning window is $\delta(n) = 1.5(Q_1 - Q_{-1})/[Q_{-1}(1 + Q_1/Q_0) + Q_0 + Q_1]$, where $Q_j = WD(n, k_m + j)$.

For a multicomponent signal, besides the cross-terms that degrade the WD, the variance of estimation of the IF of the signal's mth component increases as

$$\sigma_{\omega_m}^2 = \sigma_\nu^2 \left[2\sum_{i=1}^M A_i^2(n) + \sigma_\nu^2 \right] G_w \Big/ [4A_m^4(n) N_w T_w^2].$$

For the SM, when it produces the sum of the WD of individual components, it is equal to (9) with $A^2 = A_m^2$, and it is lower than in the WD. In the case of the SPEC a significant increase of the variance σ_ω^2 appears as a function of $\omega'(t)$, if the IF is not constant over the considered window length [29]. Thus, it will not be discussed here.

The bias of the IF does not exist in any of these distributions if the IF may be considered constant or linear within the considered window. If the IF is nonlinear, then higher order distributions achieve its local linearization [22], [23].

For the *amplitude estimation* analysis we have to know the variance of the WD with respect to the input noise. It is given by [25], [35]

$$\sigma_{WD}^2(n, k) = 4\sigma_\nu^2(2A^2(n) + \sigma_\nu^2)E_w T_w^2/N_w^2 \qquad (10)$$

where $E_w = \sum_{n=1}^{N_w} w^4(n)$. The squared amplitude of a component is estimated using the time-marginal property

$$\hat{A}^2(n) = \sum_{n=1}^{N_w} WD(n, k) \Delta\omega/2\pi. \qquad (11)$$

This summation is usually performed over a few values of the WD around the frequency where the maximum is detected, $k_m(n) - K \le k \le k_m(n) + K$, since the energy is located in a narrow region around that point. The leakage effects, which may adversely affect the precision, can be reduced using special window types, such as flat-top windows, [29]. In the case of the Hanning window and signal with slow-varying amplitude we may efficiently use $K = 1$, along with the already found displacement in frequency $\delta(n)$, so as to correct values $WD(n, k_m)$ and $WD(n, k_m \pm 1)$ and to avoid leakage.

The variance of estimated amplitude (10), (11) is

$$\sigma_A^2 = (2K + 1)\sigma_\nu^2(2A^2(n) + \sigma_\nu^2)E_w/N_w^2. \qquad (12)$$

Bias of the amplitude estimation is [35]
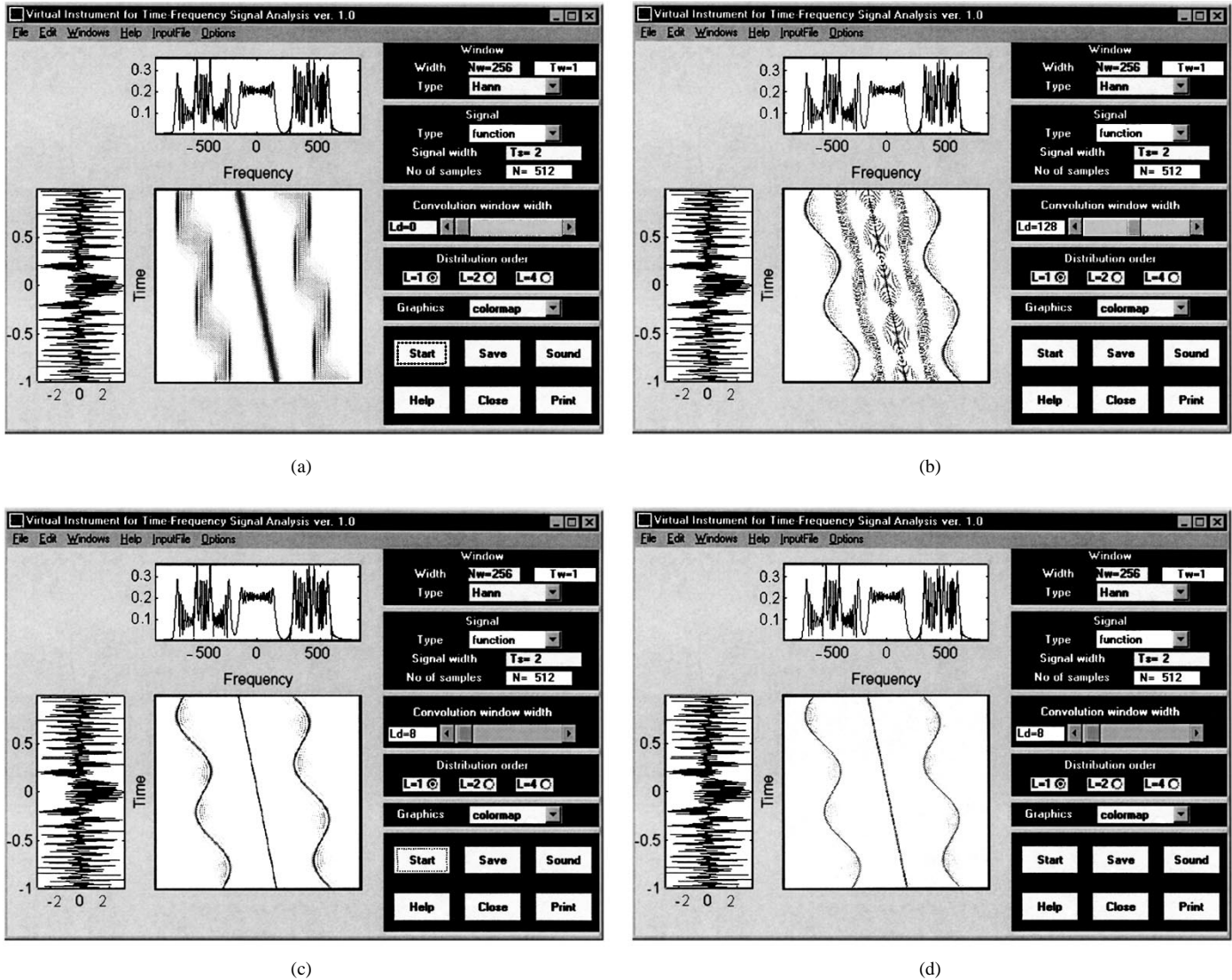
$$B_A = (2K + 1)\sigma_\nu^2/(2N_w). \qquad (13)$$

Fig. 4. Time–frequency representation of a simulated multicomponent signal: (a) spectrogram, (b) Wigner distribution, (c) S-method, and (d) fourth order L-Wigner distribution realized using the S-method. Relevant parameters are shown in the figure.

In the case of a multicomponent signal the variance is increased. For TF nonoverlaping components, $A^2(n)$ in (10) is replaced by $\sum_{m=1}^{M} A_m^2$. In the case of the SM the variance is lower since A^2 should be replaced by A_m^2, for the corresponding component.

We will use an example to illustrate the instrument performances with respect to the amplitude and the IF estimation. Consider an analytic part of the linear FM signal $x(t) = \exp(-t^2/4)\cos(15\pi t^2 + 90\pi t)$ with additive noise $\nu(t)$. The signal is sampled at $T = 1/256$, using the Hanning window of the width $T_w = 2$ with $N_w = 512$. The variance of discrete noise $\nu(nT)$ is $\sigma_\nu = 0.1$. Using $K = 1$, from (12) we obtain $\sigma_A^2 = 4.4165 \times 10^{-5}$, for the maximal amplitude $A = 1$. The bias, according to (13), is $B_A = 2.9297 \times 10^{-5}$. Therefore, the error of $A^2(n)$ estimation is within $\pm(B_A + 2\sigma_A) = 1.33 \times 10^{-2}$ with the probability of 95% for the Gaussian distribution of error. Variance of the IF is $\sigma_\omega^2 = 6.8947 \times 10^{-5}$. The frequency axis quantization error which would have had the variance $\sigma_q^2 = 0.20562$ had been reduced bellow the

order of other variances by using the interpolation method with displacement bin $\delta(n)$ [29]. Note that the maximal frequency is $\omega_m = N_w\pi/(2T_w) = 128\pi$, meaning that the relative error is very low. The above analysis is performed for the WD and the SM with $L_d = 6$. Note that in the SM we could use twice wider sampling interval. All these results have been statistically checked.

When the input noise is only the A/D quantization error then, assuming 12 bit conversion, we get $\sigma_\nu^2 = 2^{-24}/3$ for complex signals, obtained as analytic versions of the real ones. Therefore, the precision is here limited by the leakage effects for the amplitude estimation and by the quantization error for the IF estimation. Ways to control and reduce these effects are already described [29].

Consider now a two-component signal $x(t) = \exp(-t^2/4 + j(15\pi t^2 + 70\pi t))(1 + \exp(-j50\pi t)) + \nu(t)$. The variance of the WD's is twice higher than in the previous cases. The cross-terms in the WD are twice higher than the product of amplitudes of signal components. The SM with
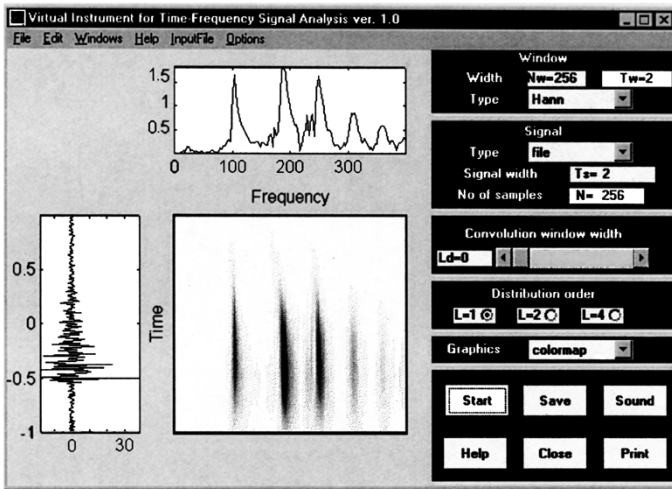
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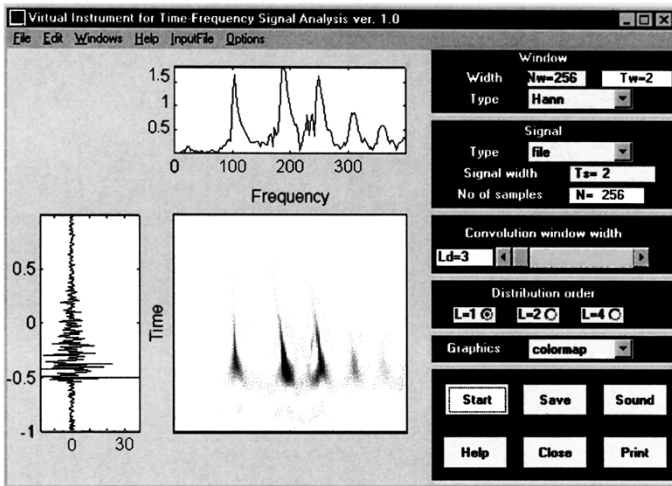
(b)

Fig. 5. Time–frequency representation of a motor vibration signal: (a) spectrogram and (b) S-method. Relevant parameters are shown in the figure.

$L_d = 6$ produces the same variances of the amplitude and IF estimation (detecting positions of local maxima) as in the one-component case.

IV. EXAMPLES

Sonar Signal Example: TF analysis of a sonar signal $x(t)$, and its delayed version $x(t-T)$ is shown in Fig. 3. The SPEC, with normalized time axis, is presented in Fig. 3(a), while the SM is presented in Fig. 3(b). The parameters are shown in the Figure. We see that by increasing L_d from 0 (SPEC) to the value of $L_d = 6$, while keeping all other parameters invariant, we significantly improve the distribution concentration, and avoid cross-terms.

Numerical Example:
Consider a multicomponent signal:

$$x(t) = e^{-j24\pi t^2} + e^{-j16\cos(2\pi t)-j24\pi(t+1.5)^2-j64\pi t}$$
$$+ e^{j16\cos(2\pi t)-j24\pi(t+1.5)^2+j64\pi t}.$$

The SPEC of this signal is shown in Fig. 4(a). One may observe that all distribution components are spread in the TF plane and that cross terms do not exist. In the WD, Fig. 4(b), all components are highly concentrated, but the cross terms are very emphatic (even completely masking the component in the middle). The SM, Fig. 4(c), produces high distribution concentration, without cross terms. Concentration may be additionally improved by increasing the distribution order. The fourth order distribution ($L = 2$, $L_d = 8$) is shown in Fig. 4(d).

Motor Vibration Signal: The motor vibration signal measured at 2000 [rev/min] is considered. Its SPEC and SM with $L_d = 3$ are shown in Fig. 5. Analysis of the IF's and amplitudes is very important in this kind of signals in order to detect engine knocking combustions, whose frequent occurrence can destroy the motor [36].

Numerical Efficiency: The SM is studied in [10]. For the parameters as in the last example, the ratios of the numbers of multiplications and additions in the WD and the SM are 5120/1152 and 2752/768, respectively. Of course, the SM requires more operations than the SPEC. These ratios are here 768/1152 and 384/768.

V. CONCLUSION

A virtual instrument for TF analysis, based on the SM, is presented. It is efficient for the analysis of multicomponent signals, since it produces a sum of the WD's of each individual signal component. The mutual component influence on the estimation of the amplitude and the IF is avoided in this way. Realization may be computationally less consuming than the WD realization, since there is no need either for the oversampling or for the analytic signal calculation. Performance analysis, along with the illustrative examples, convincingly demonstrate the improvements.

REFERENCES

[1] *Proc. IEEE,* vol. 84, Sept. 1996.
[2] M. H. Ackroyd, "Short-time spectra and time–frequency energy distribution," *J. Acoust. Soc. Amer.,* vol. 50, pp. 1229–1231, 1970.
[3] M. G. Amin and K. D. Feng, "Short-time Fourier transform using cascade filter structures," *IEEE Trans. Circuits Syst.,* vol. 42, pp. 631–641, Oct. 1995.
[4] M. G. Amin, "A new approach to recursive Fourier transform," *Proc. IEEE,* vol. 75, pp. 1357–1358, 1987.
[5] K. D. Feng, M. G. Amin, and S. Tyler, "Analysis of the recursive multiple window STFT's and spectrograms," in *Proc. IEEE IS-TFTSA,* Philadelphia, PA, Oct. 1994, pp. 72–75.
[6] F. Hlawatsch and G. F. Boudreaux-Bartels, "Linear and quadratic time–frequency signal representations," *IEEE Signal Process. Mag.,* pp. 21–67, Apr. 1992.
[7] S. Kadambe and G. F. Boudreaux-Bartels, "A comparison of the existence of 'cross terms' in the Wigner distribution and the squared magnitude of the wavelet transform and short-time Fourier transform," *IEEE Trans. Signal Processing,* vol. 40, pp. 2498–2517, Oct. 1992.

[8] K. J. R. Liu, "Novel parallel architecture for short time Fourier transform," *IEEE Trans. Circuits Syst.,* vol. 40, pp. 786–789, Dec. 1993.

[9] R. S. Orr, "The order of computation of finite discrete Gabor transform," *IEEE Trans. Signal Processing,* vol. 41, pp. 122–130, Jan. 1993.

[10] L. Stanković, "A method for time-frequency signal analysis," *IEEE Trans. Signal Processing,* vol. 42, pp. 225–229, Jan. 1994.

[11] J. Jeong and W. J. Williams, "Mechanism of the cross-terms in spectrograms," *IEEE Trans. Signal Processing,* vol. 40, pp. 2608–2613, Oct. 1992.

[12] L. E. Atlas, Y. Zhao, and R. J. Marks, II, "The use of cone shape kernels for generalized time-frequency representations of nonstationary signals," *IEEE Trans. Acoust., Speech, Signal Processing,* vol. 38, pp. 1084–1091, 1990.

[13] B. Boashash and J. B. Black, "An efficient real time implementation of the Wigner–Ville distribution," *IEEE Trans. Acoust., Speech, Signal Processing,* vol. ASSP-35, pp. 1611–1618, Nov. 1987.

[14] H. Choi and W. Williams, "Improved time-frequency representation of multicomponent signals using exponential kernels," *IEEE Trans. Acoust., Speech, Signal Processing,* vol. 37, pp. 862–871, June 1989.

[15] L. Cohen, "Time-frequency distributions—A review," *Proc. IEEE,* vol. 77, pp. 941–981, July 1989.

[16] T. A. C. M. Claasen and W. F. G. Mecklenbrauker, "The Wigner distribution: A tool for time frequency signal analysis, Parts I, II, and III," *Philips J. Res.,* vol. 35, pp. 3–6, 1980.

[17] L. Stanković, "The auto-term representation by the reduced interference distributions: The procedure for a kernel design," *IEEE Trans. Signal Processing,* vol. 44, June 1996.

[18] B. Zhang and S. Sato, "A time-frequency distribution of Cohen's class with a compound kernel and its application to speech signal processing," *IEEE Trans. Signal Processing,* vol. 42, pp. 54–64, Jan. 1994.

[19] D. Wu and J. M. Morris, "Time-frequency representations using radial butterworth kernel," in *Proc. IEEE IS-TFTSA,* Philadelphia, PA, Oct. 1994, pp. 60–63.

[20] Y. M. Zhu, F. Peyrin, and R. Goutte, "Transformation de Wigner–Ville: description d' un nouvel outil de traitement du signal et des images," *Ann. Telecomm.,* vol. 42, no. 3/4, pp. 105–117, 1987.

[21] P. Flandrin, "Some features of time-frequency representation of multicomponent signals," in *Proc. IEEE Int. Conf. Acoustics, Speech, Signal Processing,* 1984, pp. 41B.4.1.–4.1.

[22] L. Stanković, "A method for improved energy concentration in the time-frequency analysis of multicomponent signals using the L-Wigner distribution," *IEEE Trans. Signal Processing,* vol. 43, pp. 1262–1269, May 1995.

[23] ——, "Multitime definition of the Wigner higher order distribution: L-Wigner distribution," *IEEE Signal Processing Lett.,* vol. 1, pp. 106–109, July 1994.

[24] D. Chester, "Discrete Wigner implementations," in *Proc. Int. Symp. Circuits Syst.,* San Jose, CA, May 1986, pp. 38–41.

[25] L. Stanković, V. Ivanović, and Z. Petrović, "Unified noise analysis in the spectrogram and Wigner distribution," *Ann. Telecom.,* vol. 51, pp. 585–594, Nov./Dec. 1996.

[26] M. Sun, C. C. Li, L. N. Sekhar, and R. J. Sclabassi, "Efficient computation of the discrete pseudo Wigner distribution," *IEEE Trans. Acoust., Speech, Signal Processing,* vol. 37, pp. 1735–1741, Nov. 1989.

[27] F. Peyrin and R. Prost, "A unified definition for the discrete-time, discrete frequency, and discrete time/frequency Wigner distributions," *IEEE Trans. Acoust., Speech, Signal Processing,* vol. 34, pp. 858–867, Aug. 1986.

[28] *Instrumentation Reference and Catalogue,* National Instruments, 1996.

[29] G. Andria, M. Savino, and A. Trotta, "Application of the Wigner–Ville distribution to measurement of transient signals," *IEEE Trans. Instrum. Meas.,* vol. 43, pp. 187–193, Apr. 1994.

[30] B. Boashash, "Estimating and interpreting the instantaneous frequency of a signal—Part 1: Fundamentals," *Proc. IEEE,* vol. 80, pp. 519–538, Apr. 1992.

[31] S. Stanković and L. Stanković, "An architecture for the realization of a system for time-frequency signal analysis," *IEEE Trans. Circuits Syst.,* vol. 44, pp. 600–604, July 1997.

[32] B. Boashash and B. Ristić, "Polynomial time-frequency and time-varying higher order spectra: Application to the analysis of multicomponent FM signals and to the treatment of multiplicative noise," *Signal Process.,* vol. 67, pp. 1–23, 1998.

[33] P. Goncalves and R. G. Baraniuk, "Pseudo affine Wigner distributions: definition and kernel formulation," *IEEE Trans. Signal Processing,* vol. 46, pp. 1505–1516, June 1998.

[34] V. Katkovnik and L. Stanković, "Instantaneous frequency estimation using the Wigner distribution with varying and data-driven window length," *IEEE Trans. Signal Processing,* vol. 46, pp. 2315–2325, Sept. 1998.

[35] L. Stanković and S. Stanković, "On the Wigner distribution of the discrete-time noisy signals with application to the study of quantization effects," *IEEE Trans. Signal Processing,* vol. 42, pp. 1863–1867, July 1994.

[36] L.Stanković and J. F. Böhme, "Time-frequency analysis of multiple resonances in combustion engine signals," *Signal Process.,* accepted for publication.

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